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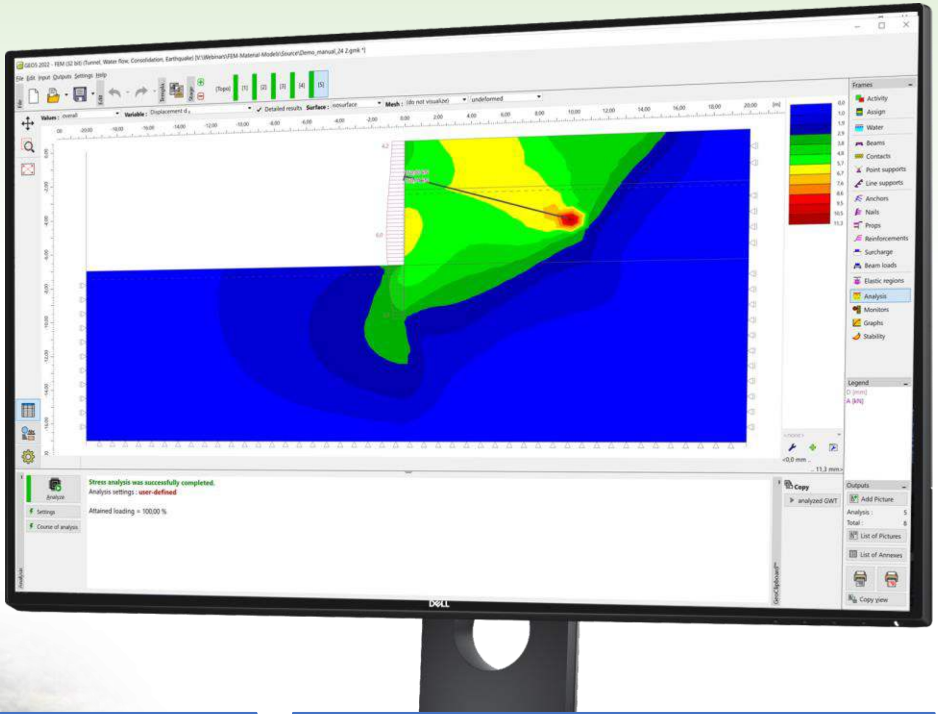
GEO5

Material models in GEO5 FEM



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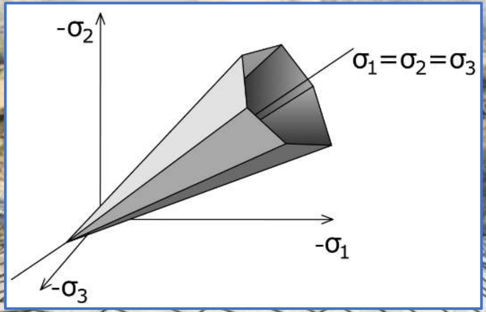
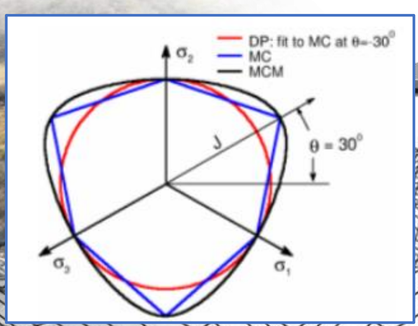
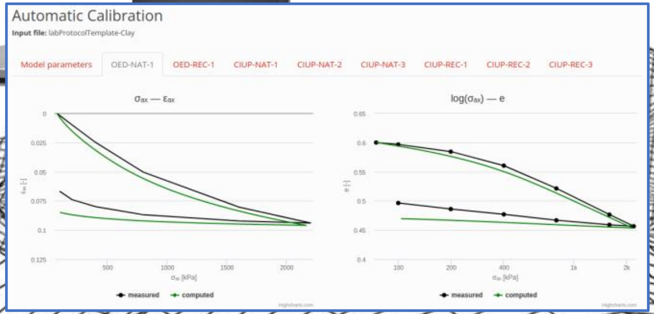






Outline

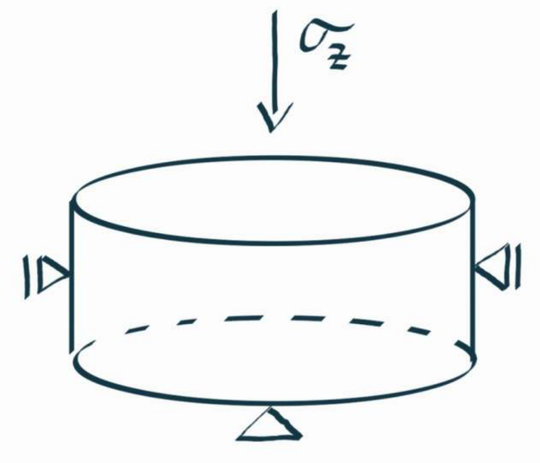
- Laboratory tests
  - Oedometric test
  - Triaxial shear test (CIUP, CID)
- Elastic models
- Elasto-plastic material models
  - Hardening, softening
- Critical state models, Cam clay, hypoplastic clay
- Stiffness depending on depth
- Geostatic stress, K_0-procedure, influence of Poisson ratio
- Calibration

Laboratory tests

Laboratory tests

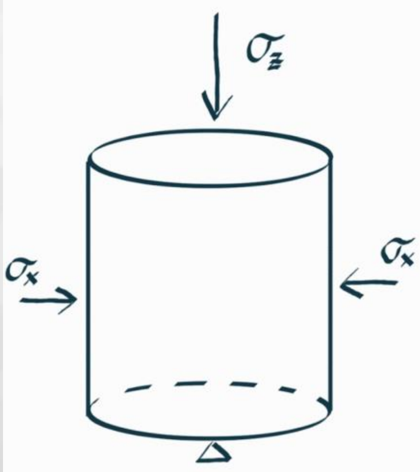
Oedometer test

(1D constrained compression)



Triaxial shear test

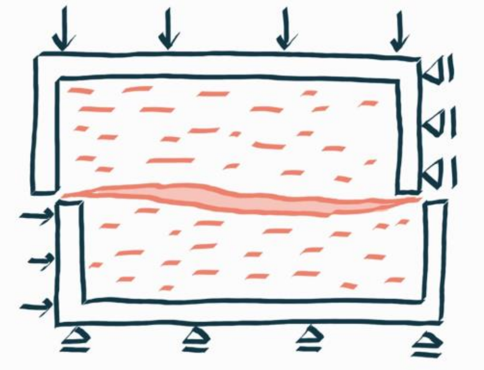
Drained (CID), undrained (CIUP)



Why these two laboratory tests?

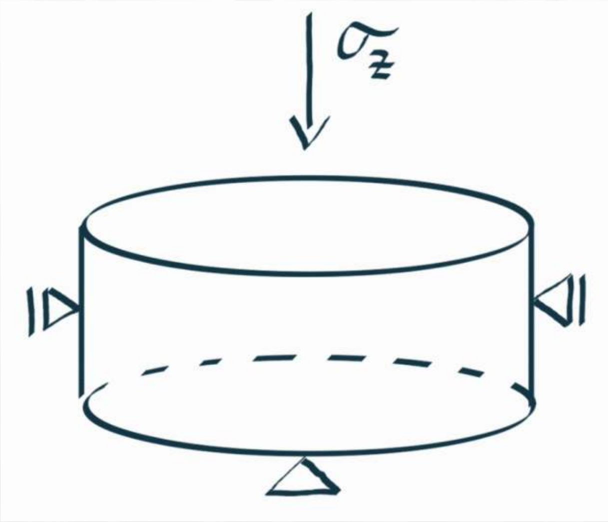
Homogeneous stress and strain.

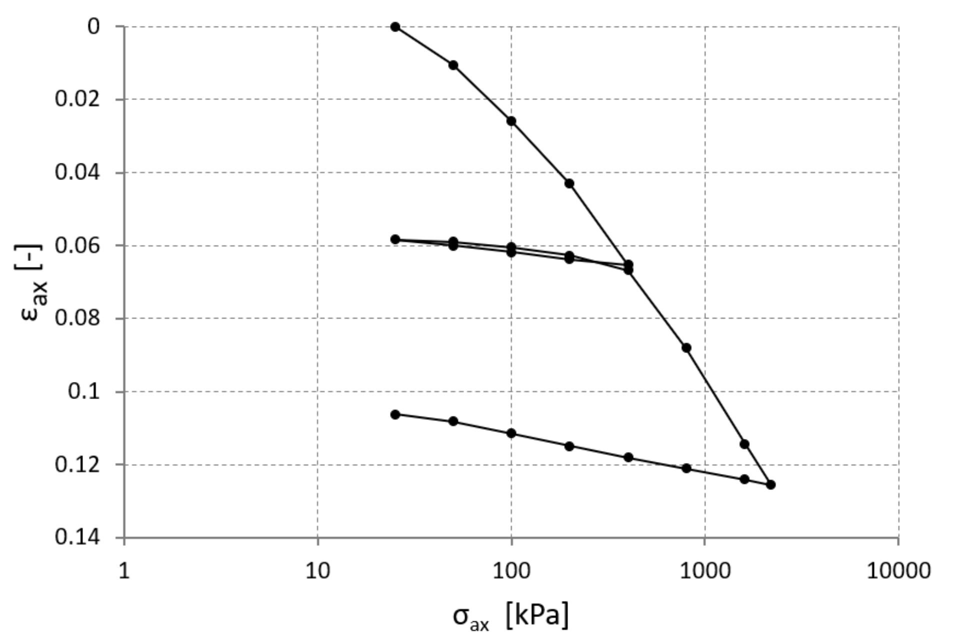
Compare to *direct shear test*.

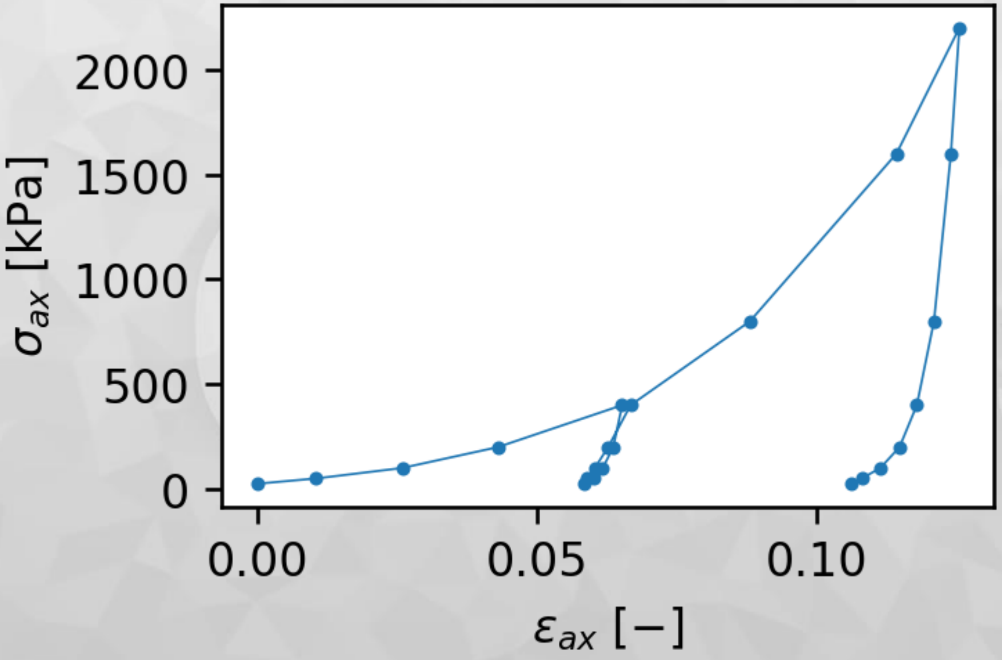


Oedometric test

- 1D constrained compression
- $\varepsilon_x = \varepsilon_y = 0$
- $\sigma_x = \sigma_y$ unknown
- σ_z, ε_z known (prescribed or measured)

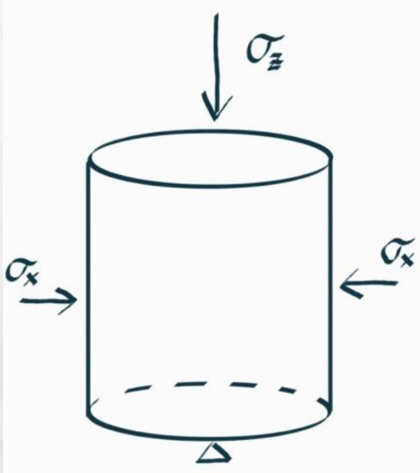


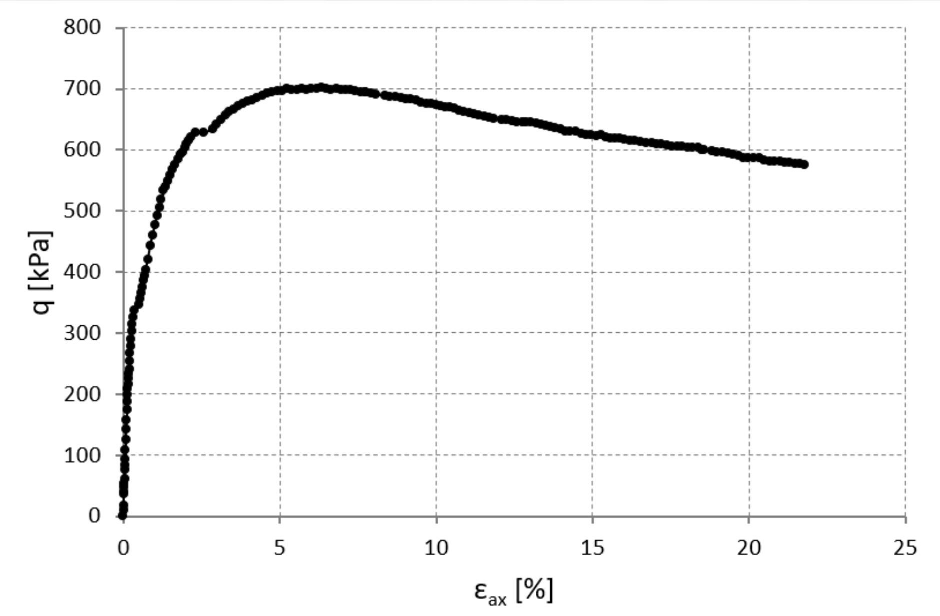


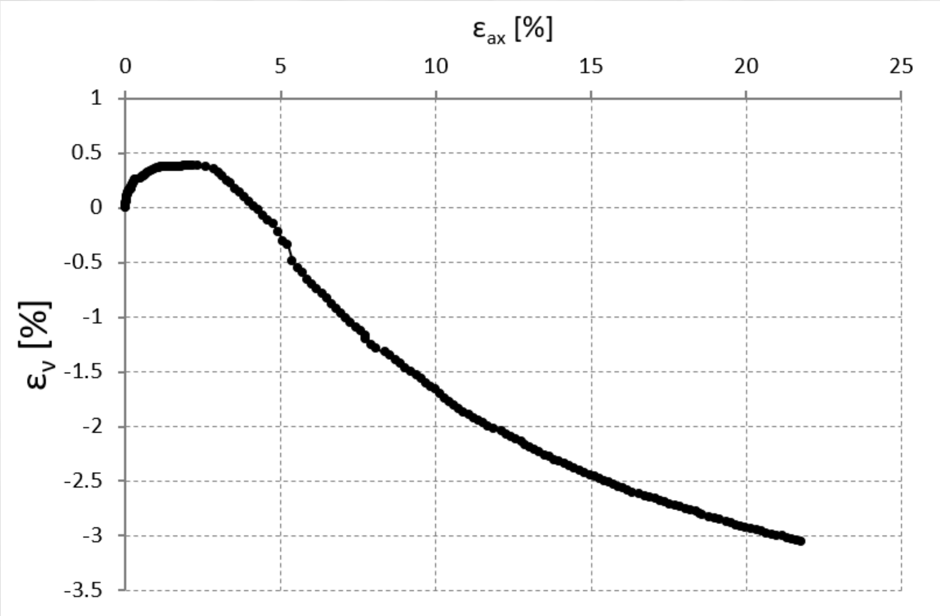


Drained triaxial shear test (CID)

- Drained test
  - Pore pressure u is constant
  - Lateral stress $\sigma_x = \sigma_y$ is constant
  - Volumetric deformation is measured

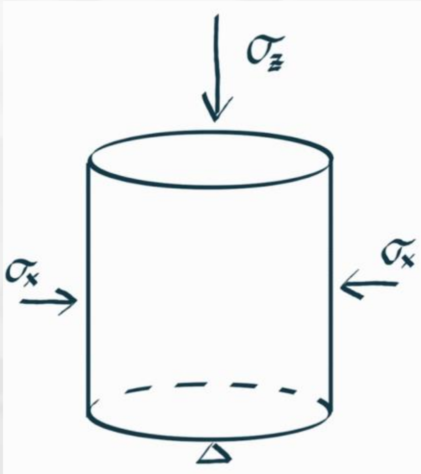


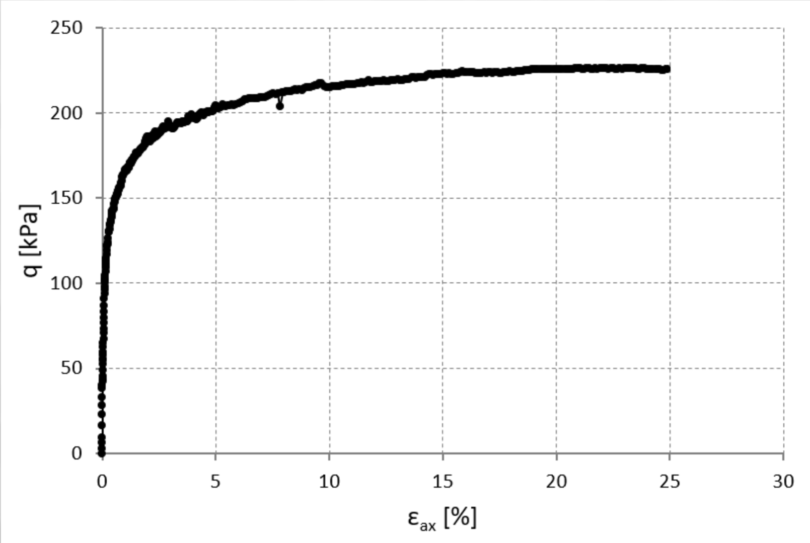


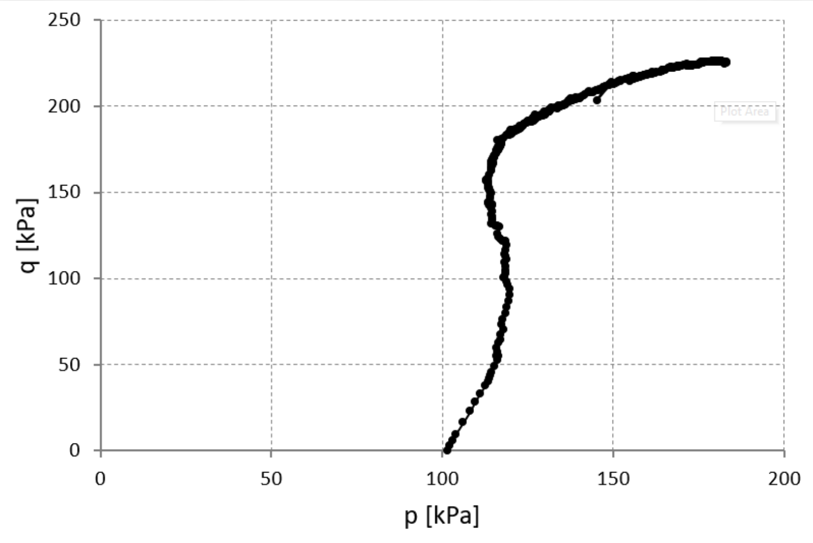


Undrained triaxial shear test (CIUP)

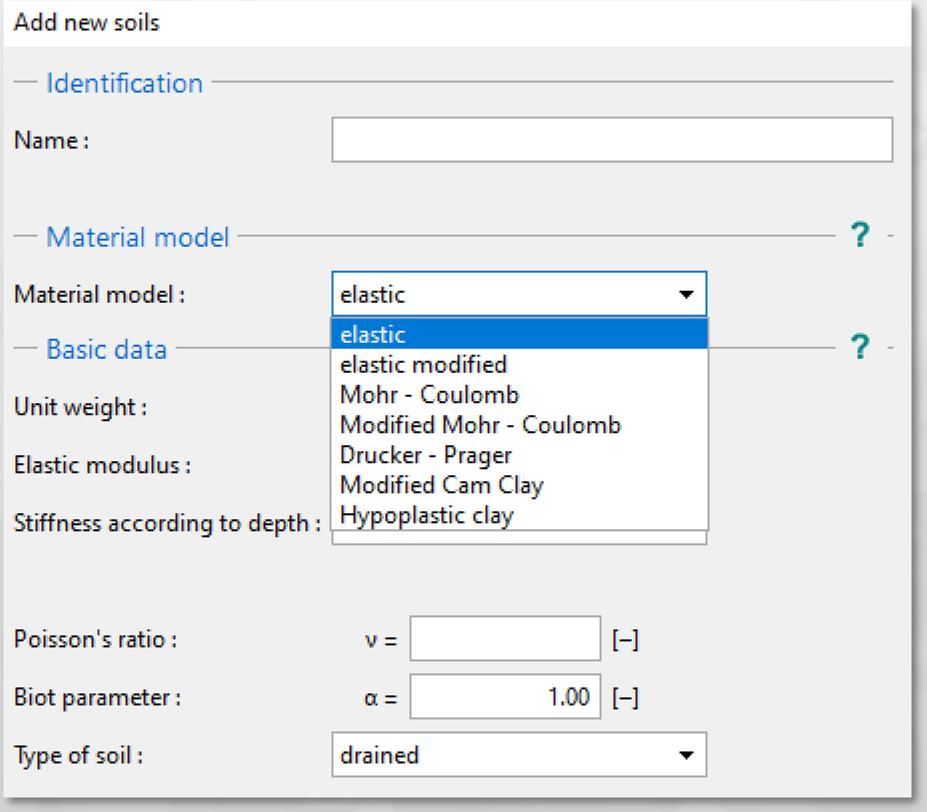
- Undrained test
  - Water does not flow out of the specimen
  - Pore pressure field is homogenous
  - Pore pressure is measured
  - Volumetric strain $\varepsilon_v = 0$
  - Simulated with prescribed displacement







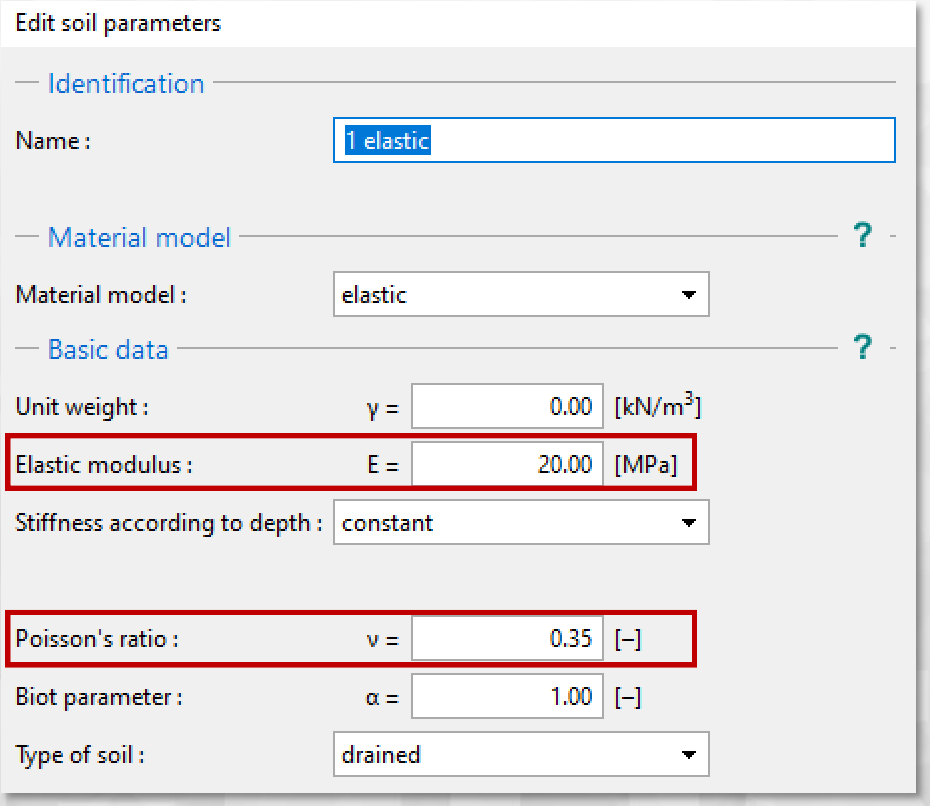
Elastic material models



Elastic material model

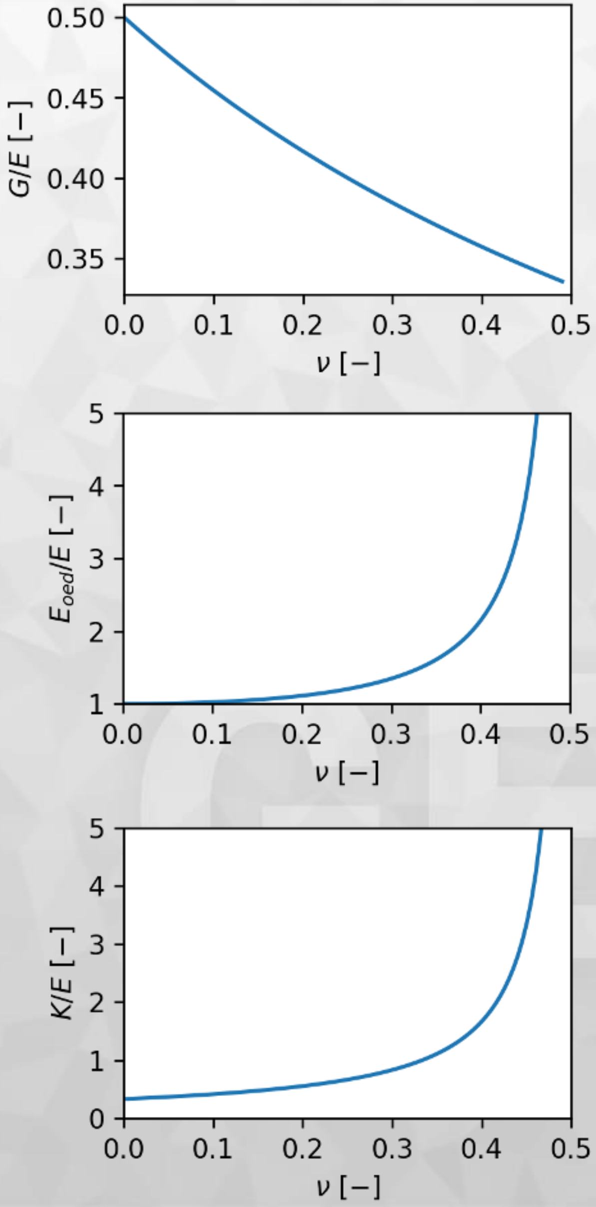
- Linear elasticity, Hooke's law
  - $\sigma_{ij} = D_{ijkl}\varepsilon_{kl}$
- Isotropic material
  - Elastic operator D_{ijkl} depends on two elastic material parameters
  - Young's modulus E
  - Poisson's ratio ν

  - Other elastic constants are also expressed in terms of E and ν.



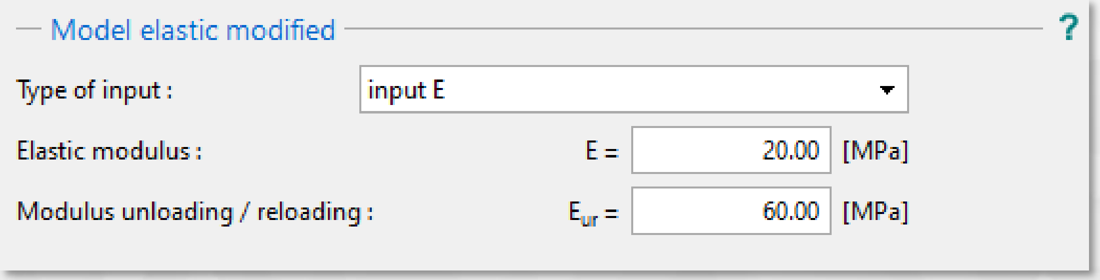
Elastic material model

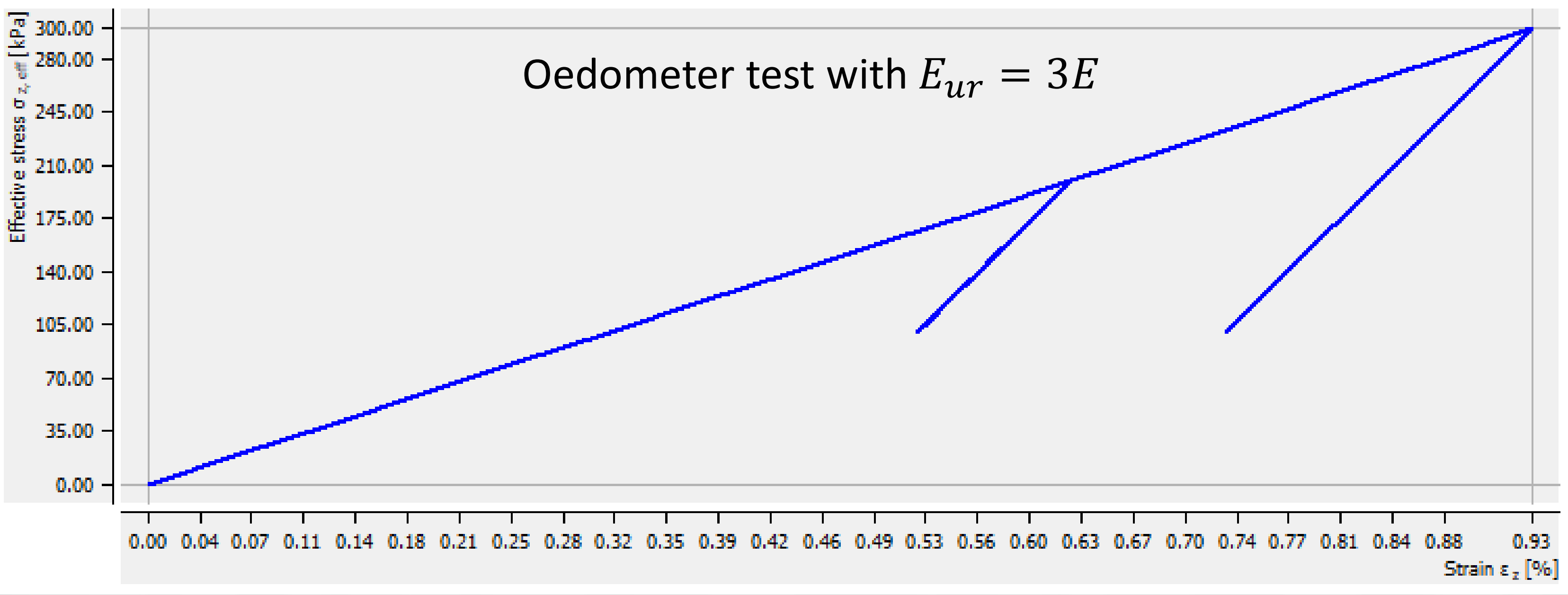
- Shear modulus
  - $G = \frac{1}{2(1+\nu)} E$
- Oedometric modulus
  - P-wave modulus M
  - $E_{oed} = \frac{(1-\nu)}{(1+\nu)(1-2\nu)} E$
- Bulk modulus
  - $K = \frac{1}{3(1-2\nu)} E$



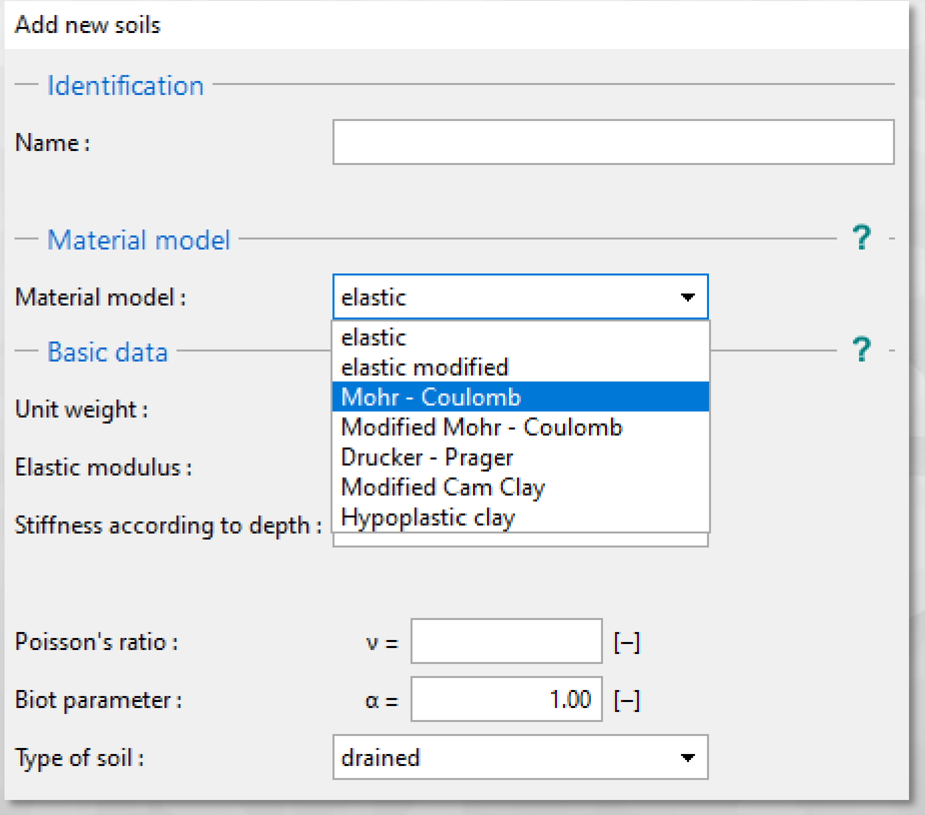
Modified elastic model

- Primary loading:
  - E … elastic modulus
- Unloading, reloading:
  - E_{ur} … unloading-reloading modulus



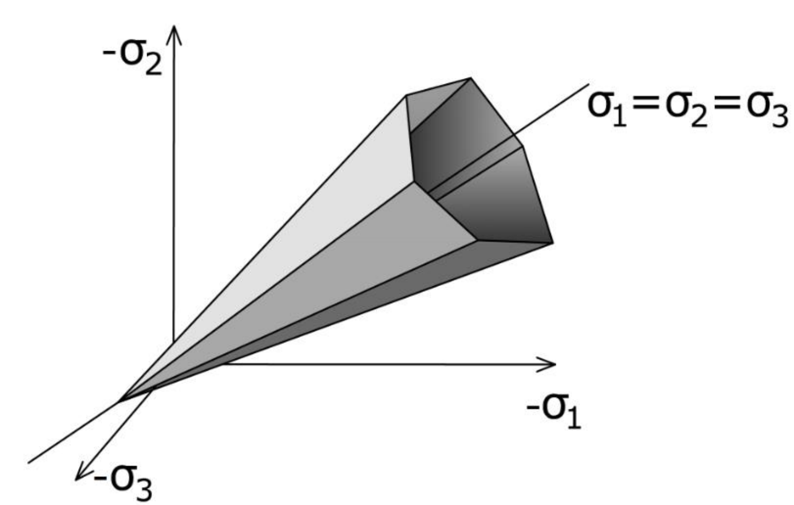


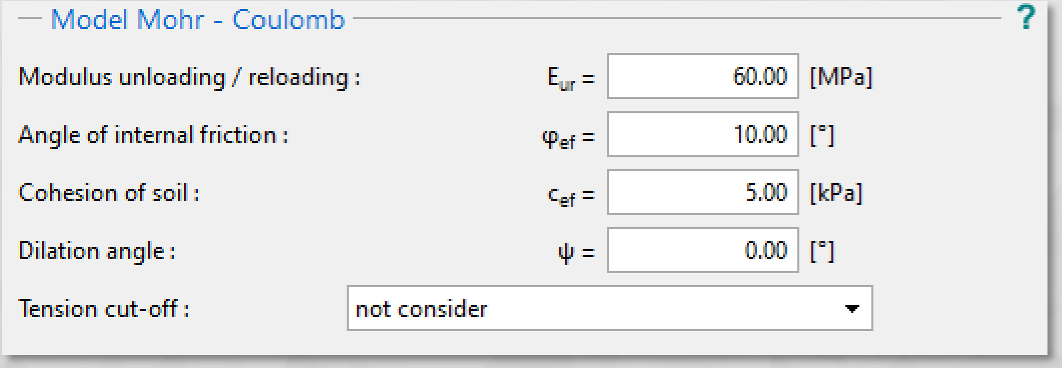
Elasto-plastic material models

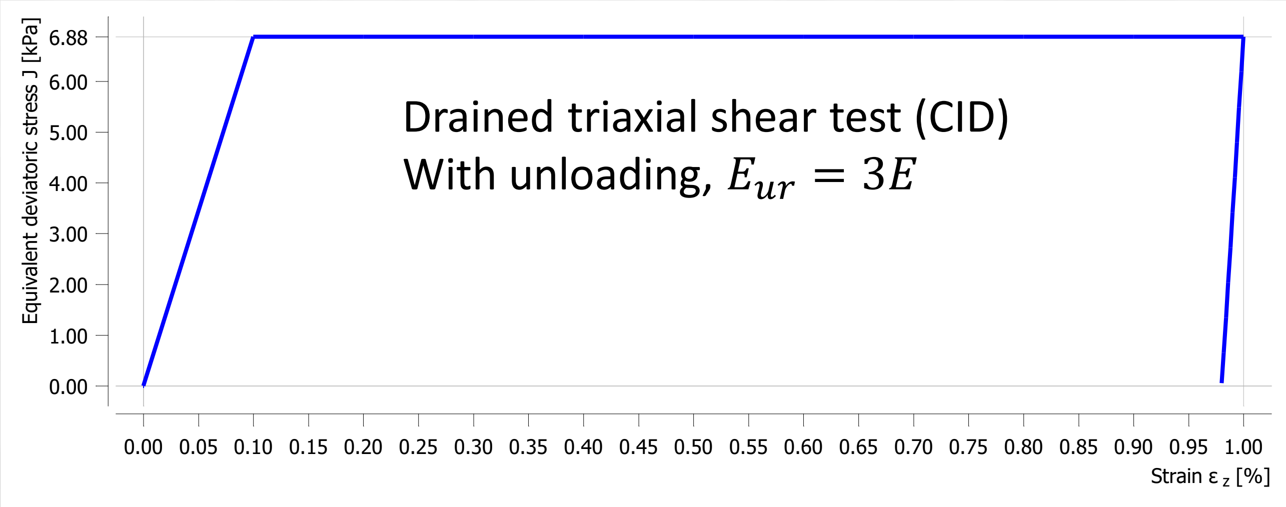


Mohr-Coulomb model

- Within elastic domain
  - Linear response
- Yield criterion – six planes
  - $\pm(\sigma_3 - \sigma_1) = (\sigma_3 + \sigma_1)\sin\varphi + 2c\cos\varphi$
  - $\pm(\sigma_3 - \sigma_2) = (\sigma_3 + \sigma_2)\sin\varphi + 2c\cos\varphi$
  - $\pm(\sigma_2 - \sigma_1) = (\sigma_2 + \sigma_1)\sin\varphi + 2c\cos\varphi$







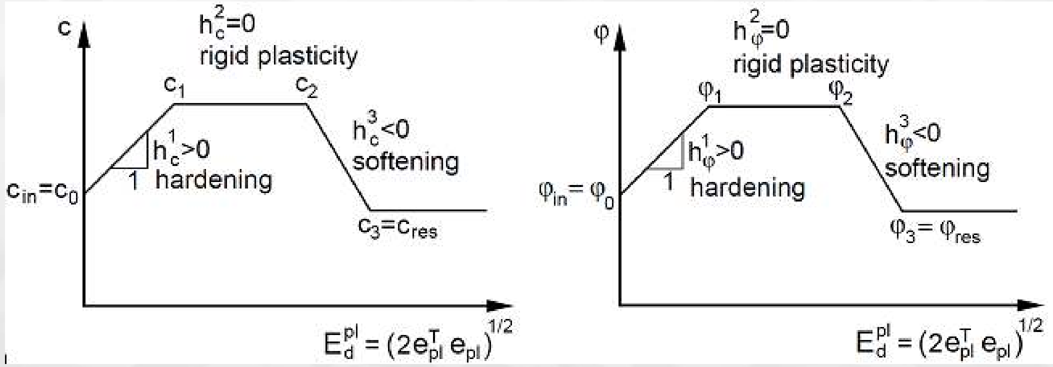
Modified Mohr-Coulomb model

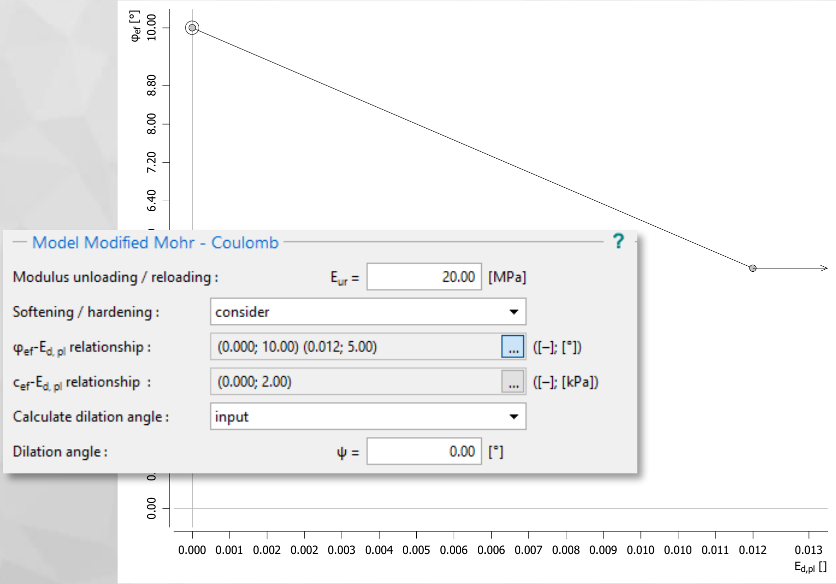
Elastic domain evolves

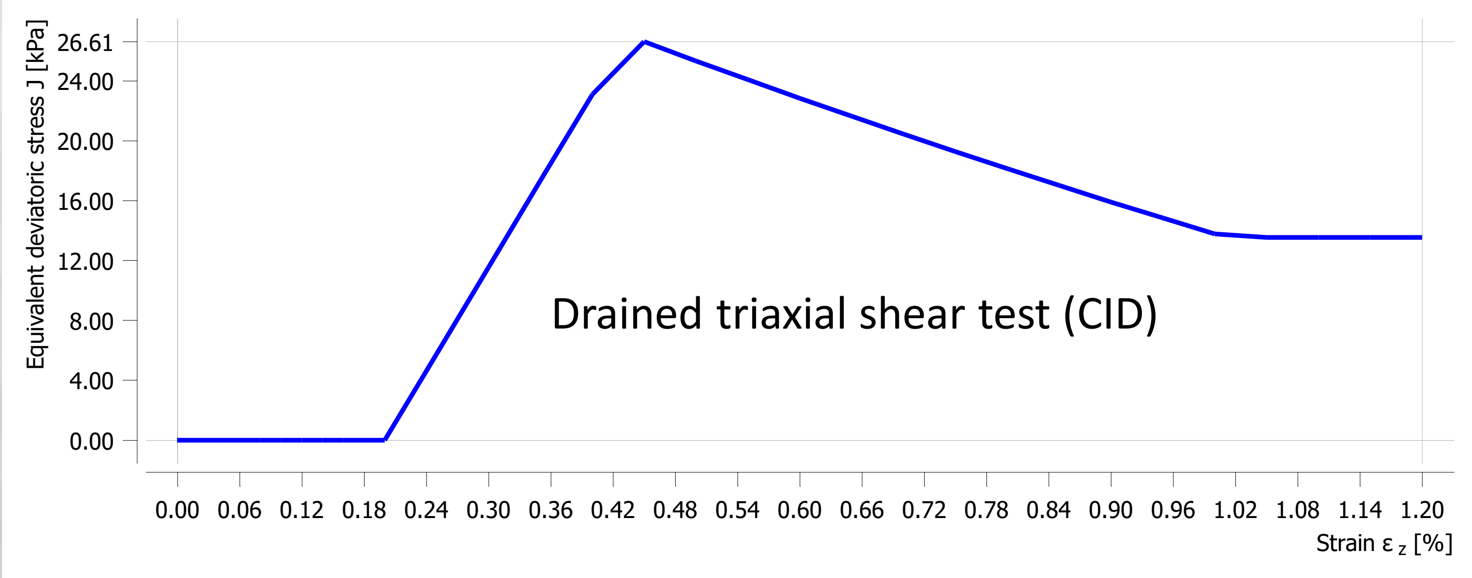
Angle of internal friction $\varphi(E_d^{pl})$

Cohesion $c(E_d^{pl})$

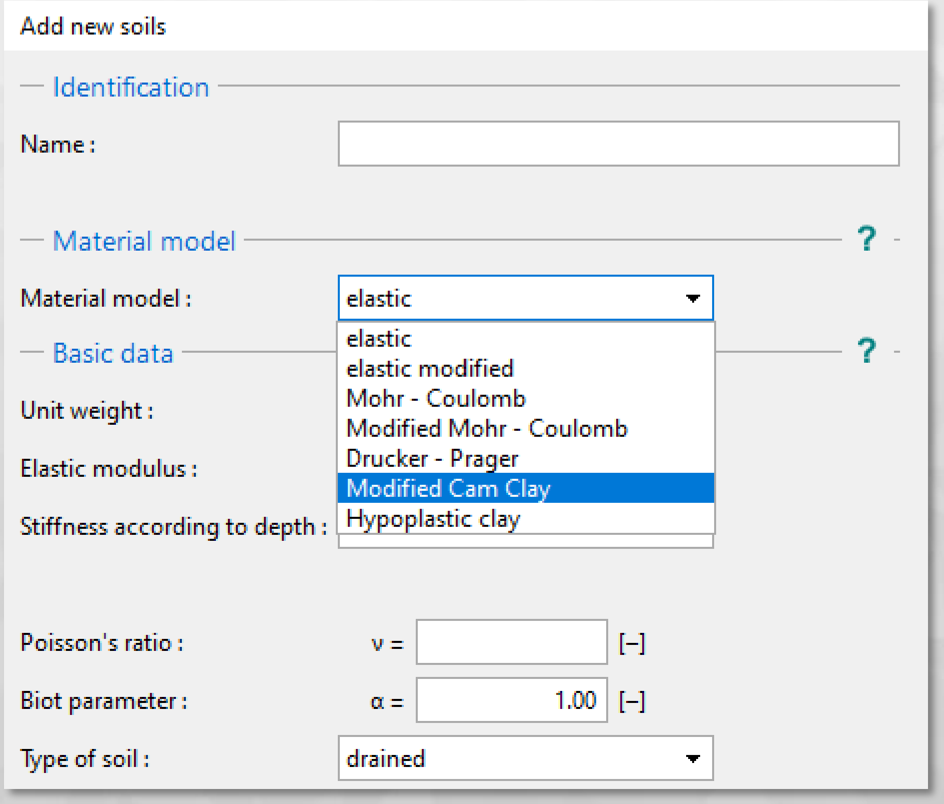
Piecewise linear functions





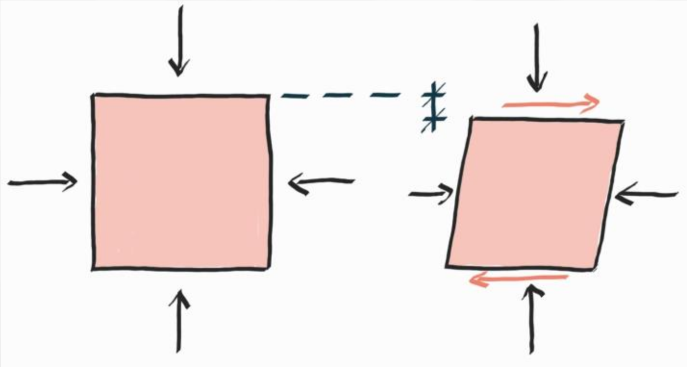


Critical state elasto-plastic material models

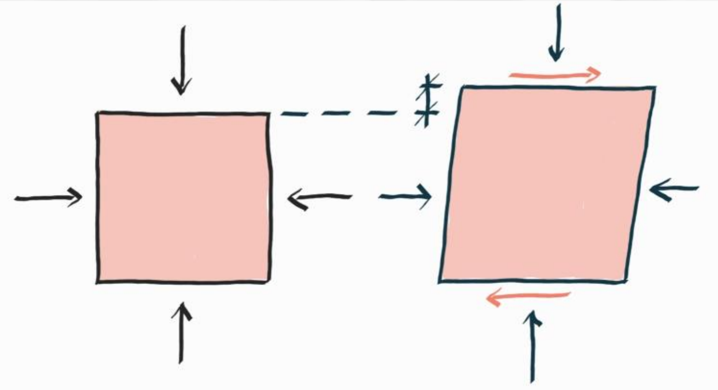


Concept of soil's state

- Loose soil contracts when sheared



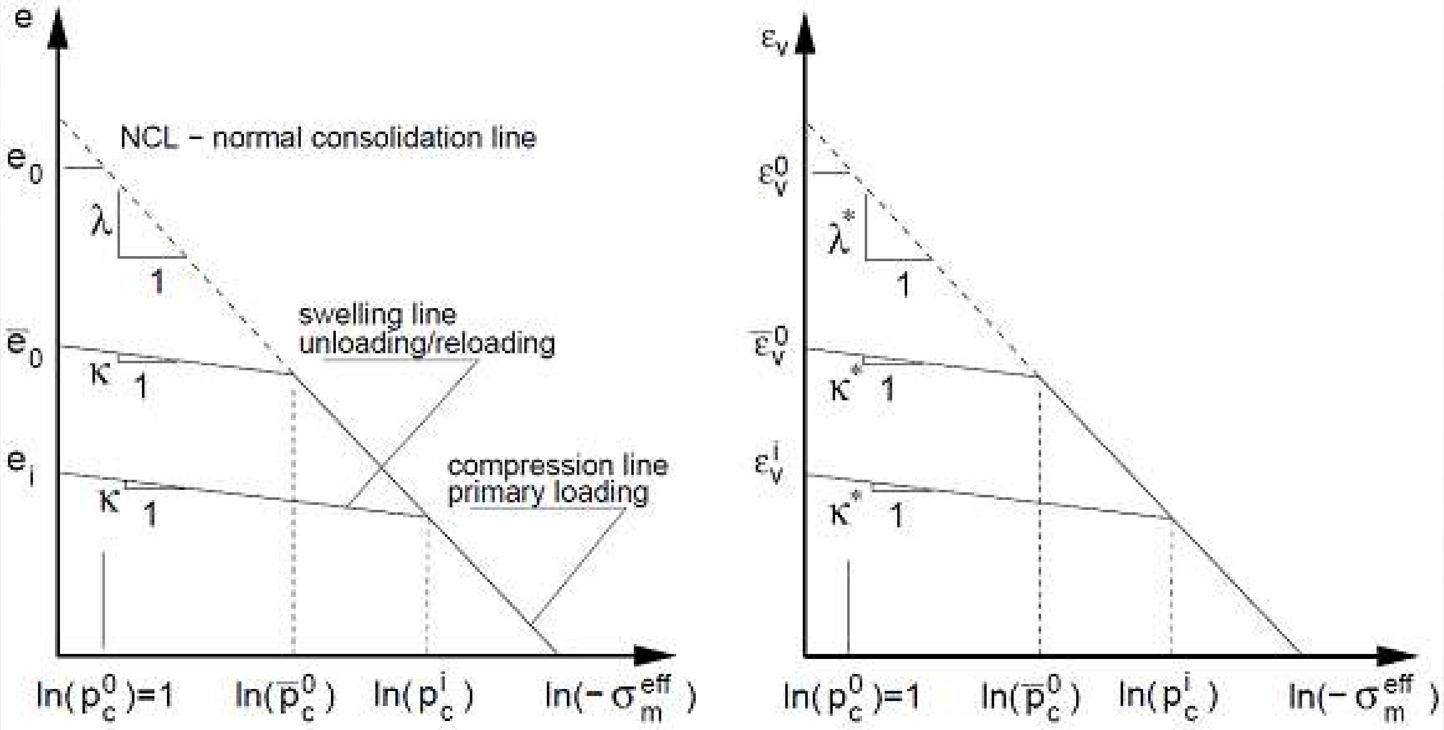
- Dense soil dilates when sheared



- The *state parameter* quantifies the soil's "density"
  - Preconsolidation pressure p_c
  - Void ratio e

Diagram of isotropic compression

- $e = e_0 - \lambda \ln(-\sigma_m)$
- $e = \bar{e}_0 - \kappa \ln(-\sigma_m)$
- $\bar{e}_0$ depends on p_c



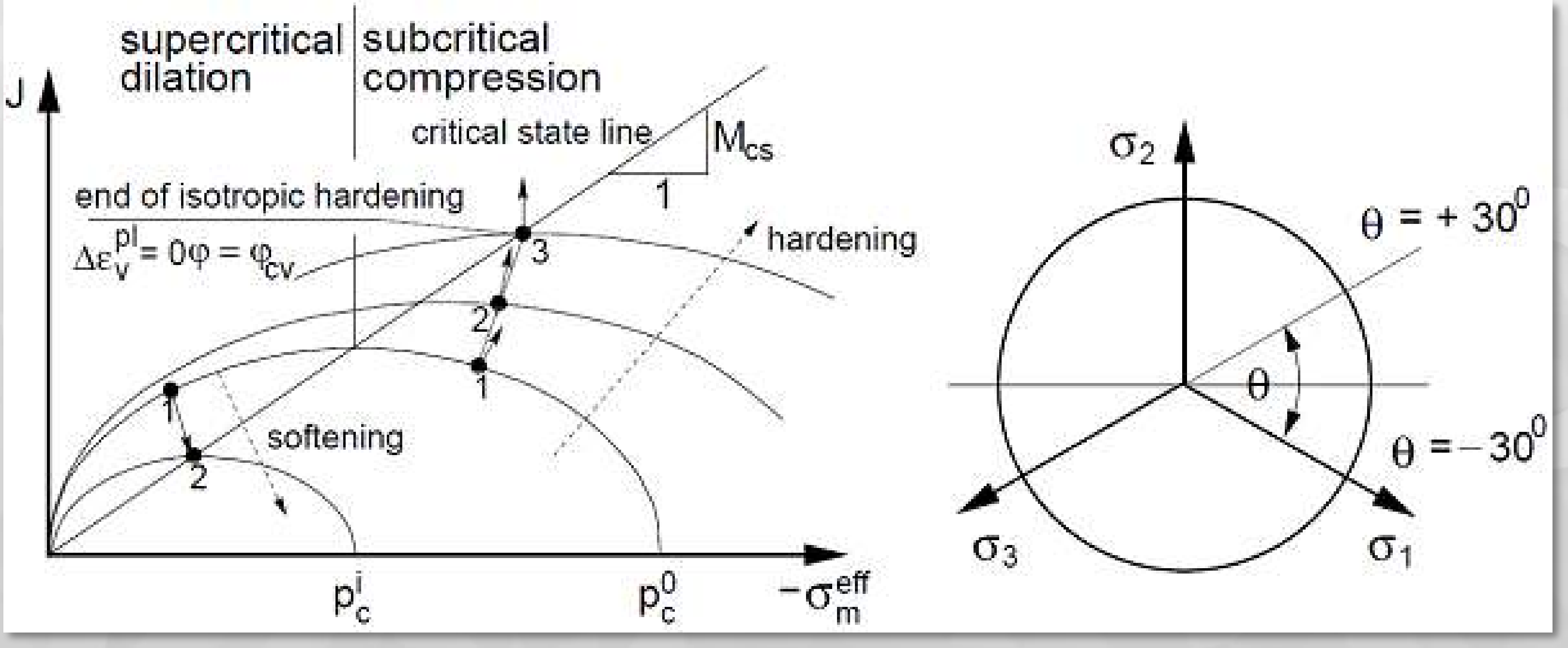
- Tangent bulk modulus
- $K_t = \frac{\sigma_m}{\kappa}$

Modified Cam clay model

- Slope of critical state line
  - $M_{cs} = \dfrac{2\sqrt{3}\sin(\varphi_{cv})}{3 - \sin(\varphi_{cv})}$
- Overconsolidation ratio
  - $OCR = \dfrac{-\sigma_m}{p_c}$

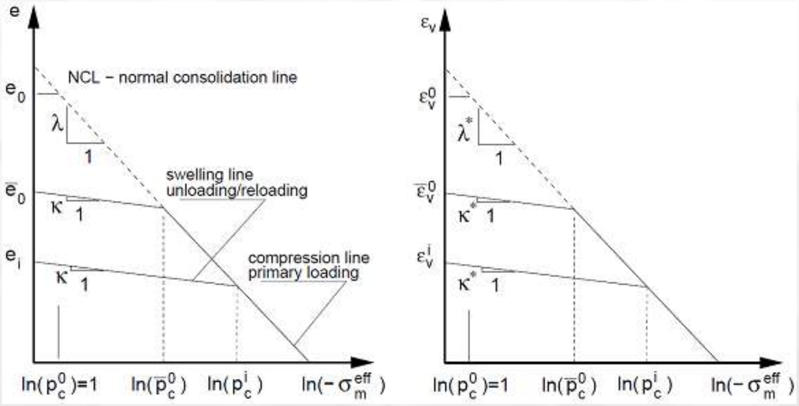
Model Modified Cam Clay ?

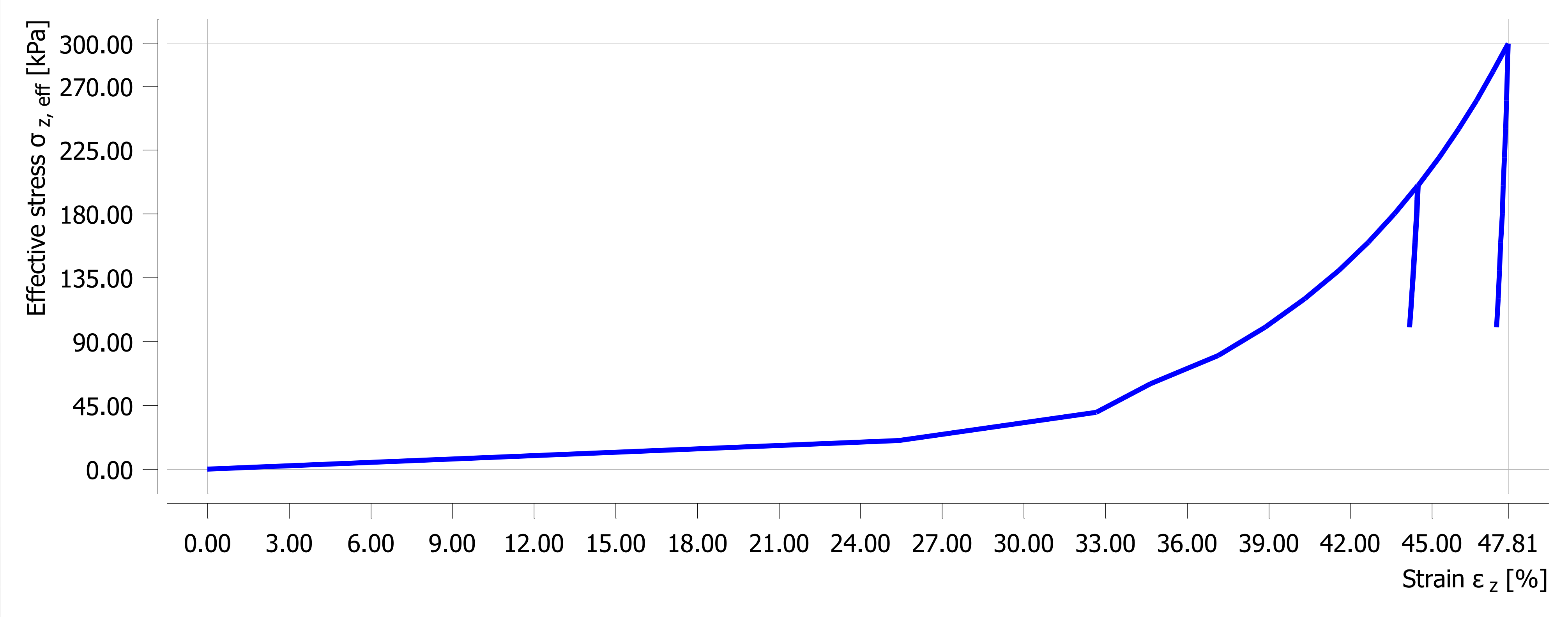
Kappa line :	κ =	0.0210	[–]
Slope of NCL :	λ =	0.2700	[–]
Slope of critical line :	M_{cs} =	0.620	[–]
Initial void ratio :	e_0 =	2.30	[–]
Overconsolidation ratio :	OCR =	1.00	[–]
Initial preconsolidation stress :	calculate		



Modified Cam clay

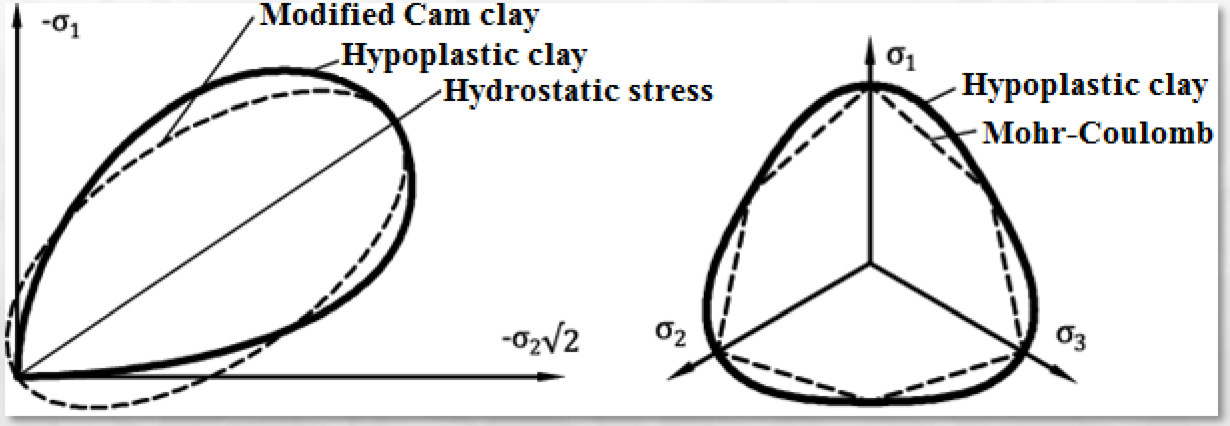
- Oedometer test with $OCR = 1$

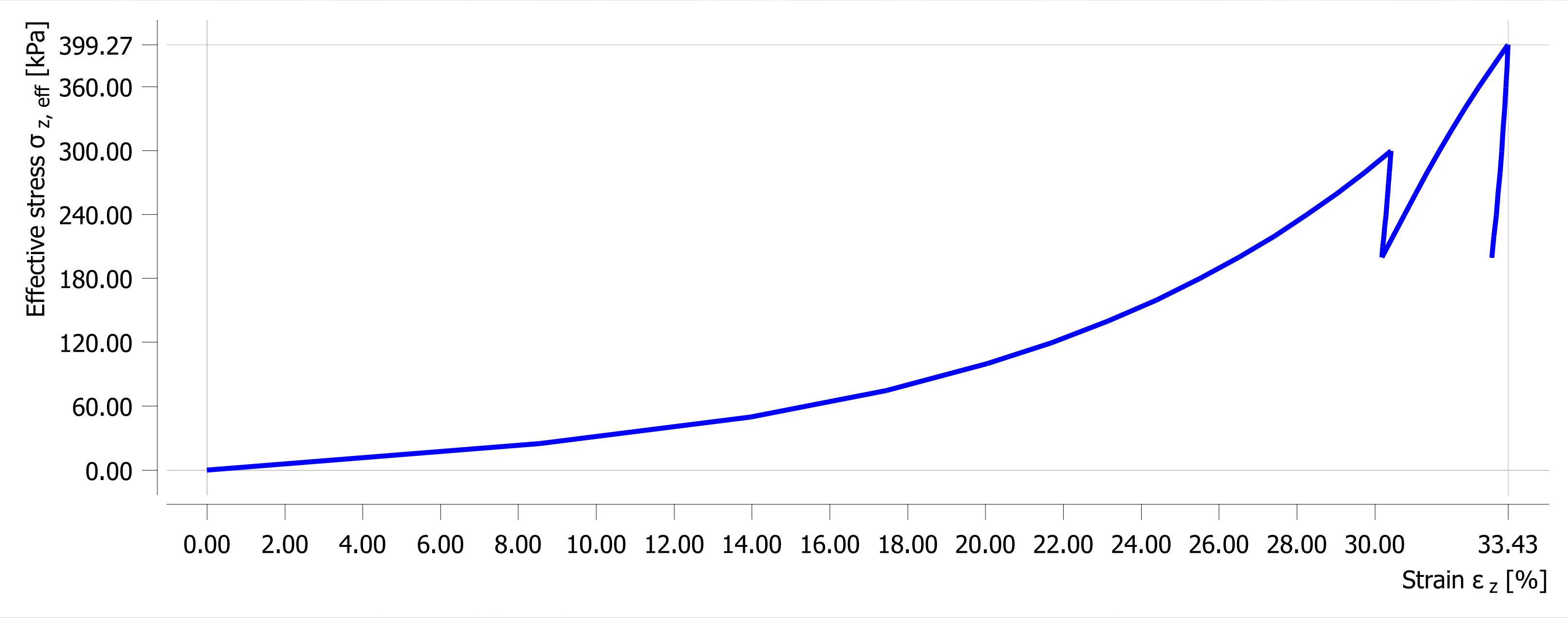




Hypoplastic clay

- Resembles Mohr-Coulomb for overconsolidated soils
- Prevents tension that Cam clay allows





Geostatic stress, K_0 procedure

Geostatic stress in 1st stage

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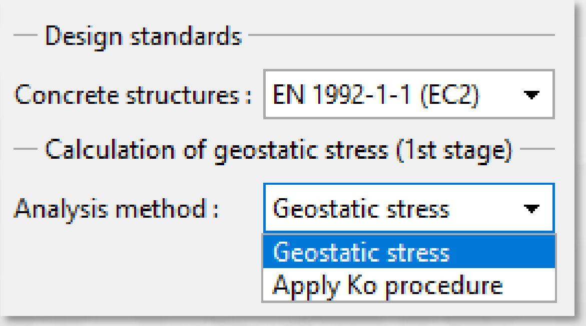


Geostatic stress

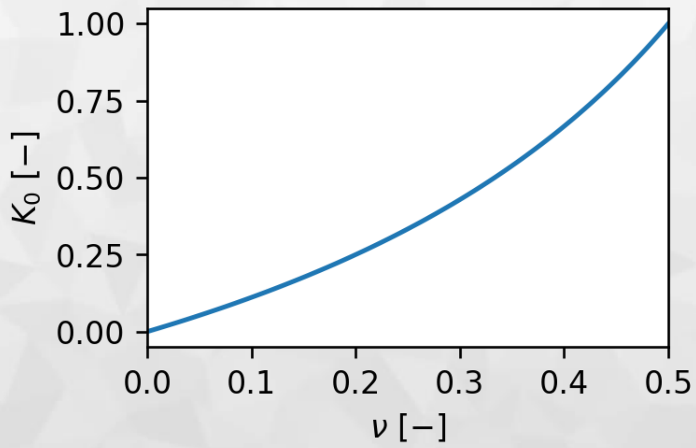
- Standard FEM analysis
- Loaded by self weight
- Displacement are zeroed

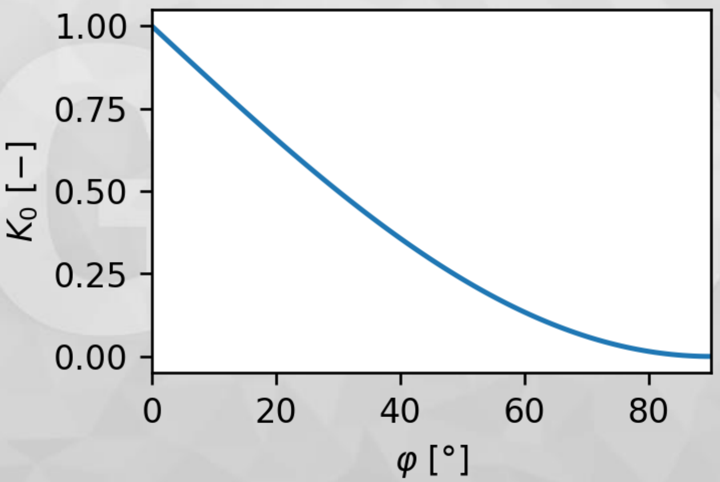
K_0 procedure

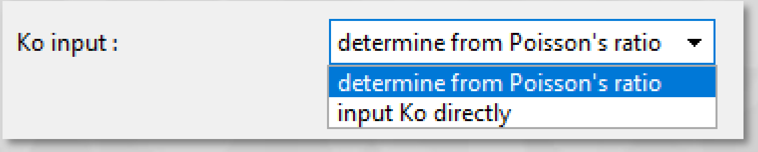
- Vertical stress $\sigma_z = \gamma h$
- Horizontal stress $\sigma_x = \sigma_y = K_0 \sigma_z$
- Shear stress $\tau_{xy} = \tau_{xz} = \tau_{yz} = 0$
- Equilibrium and strength condition may not be satisfied

Coefficient of lateral pressure K_0

- Ratio of horizontal to vertical stress
  - $K_0 = \frac{\sigma_x}{\sigma_z}$
- Theory of elasticity for $\varepsilon_x = \varepsilon_y = 0$
  - $K_0 = \frac{\nu}{1 - \nu}$
- Jaky's formula
  - Perfect plasticity
  - Cohesionless soils
  - $K_0 = 1 - \sin(\varphi)$

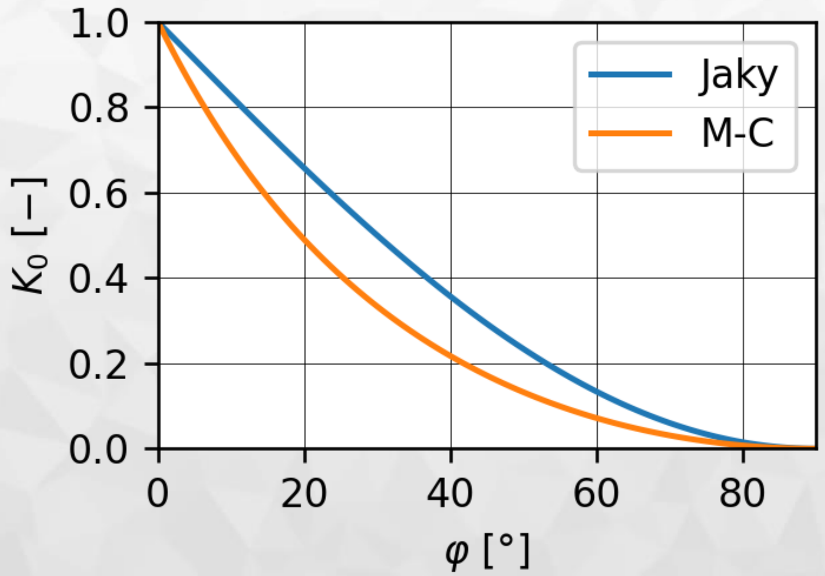


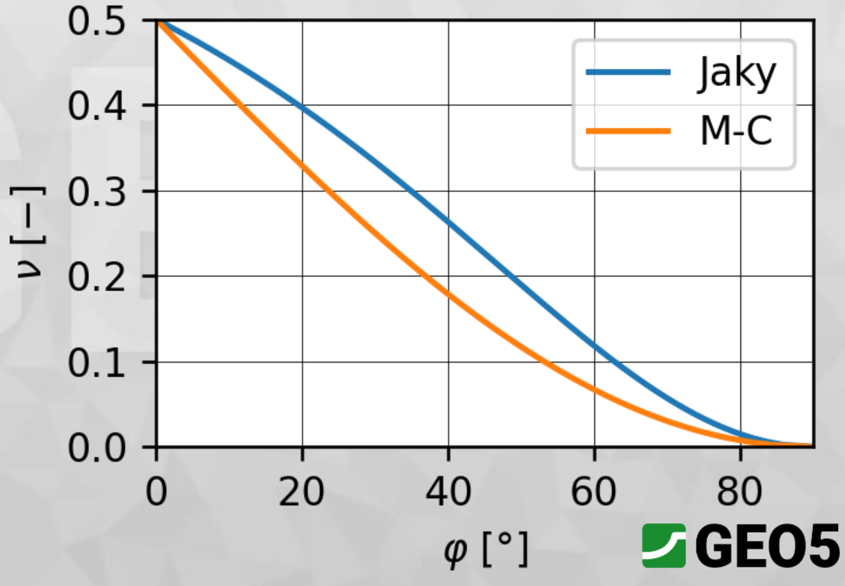




Coefficient of lateral pressure K_0

- Mohr-Coulomb failure criterion
  - $\sigma_3 - \sigma_1 = (\sigma_3 + \sigma_1)\sin\varphi + 2c\cos\varphi$
- Minimal value of K_0 for cohesionless soil
  - $K_{0,MC} = \frac{\sigma_x}{\sigma_z} = \frac{1-\sin\varphi}{1+\sin\varphi}$
- Minimal value of Poisson ratio



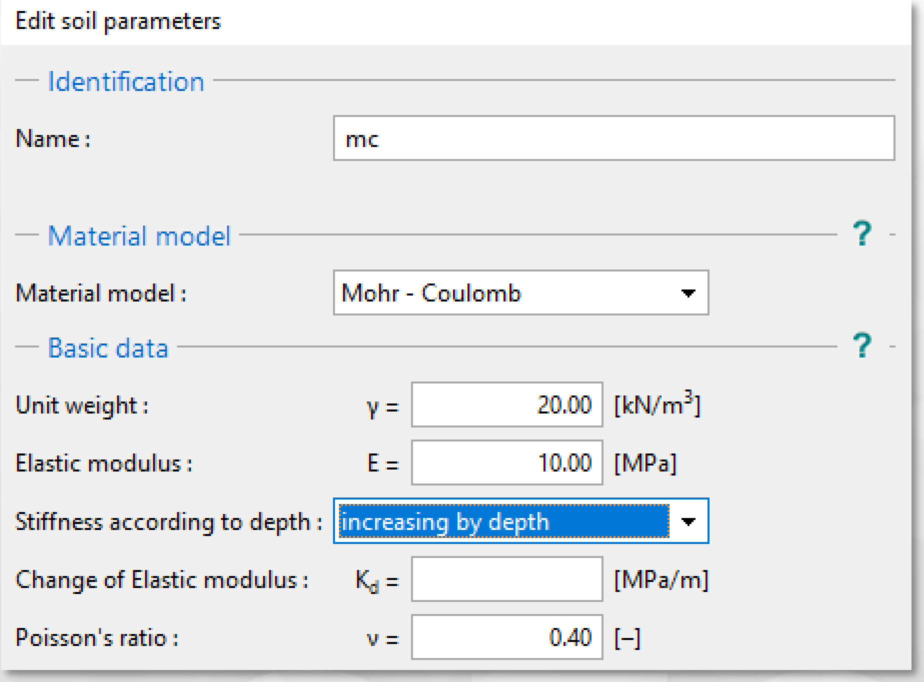


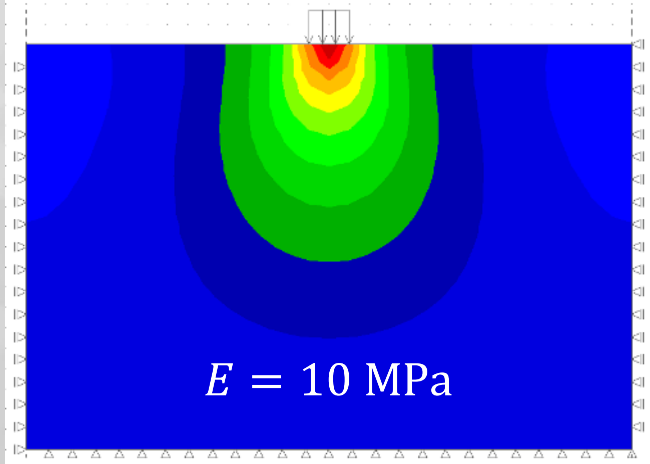
Stiffnes depending on depth

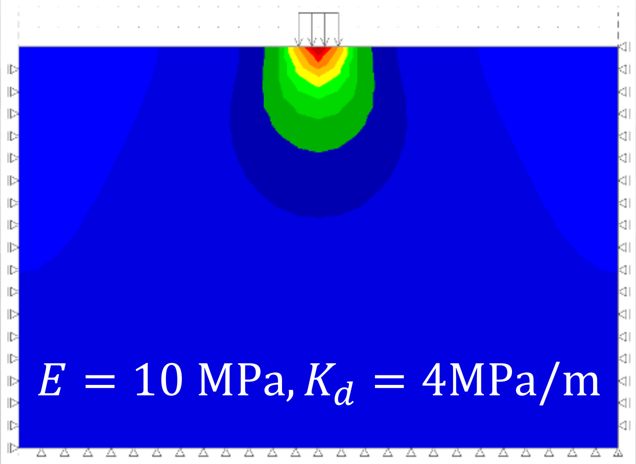
Stiffness depending on depth

- Defined at material level
  - $E(h) = E + K_d\, h$

- First guess of K_d
  - $\sigma_z = \gamma h$
  - $\sigma_x = K_0 \sigma_z$
  - $\sigma_m = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$
  - Cam clay: $K = \frac{\sigma_m}{\kappa}$
  - $E = E(K, \nu)$







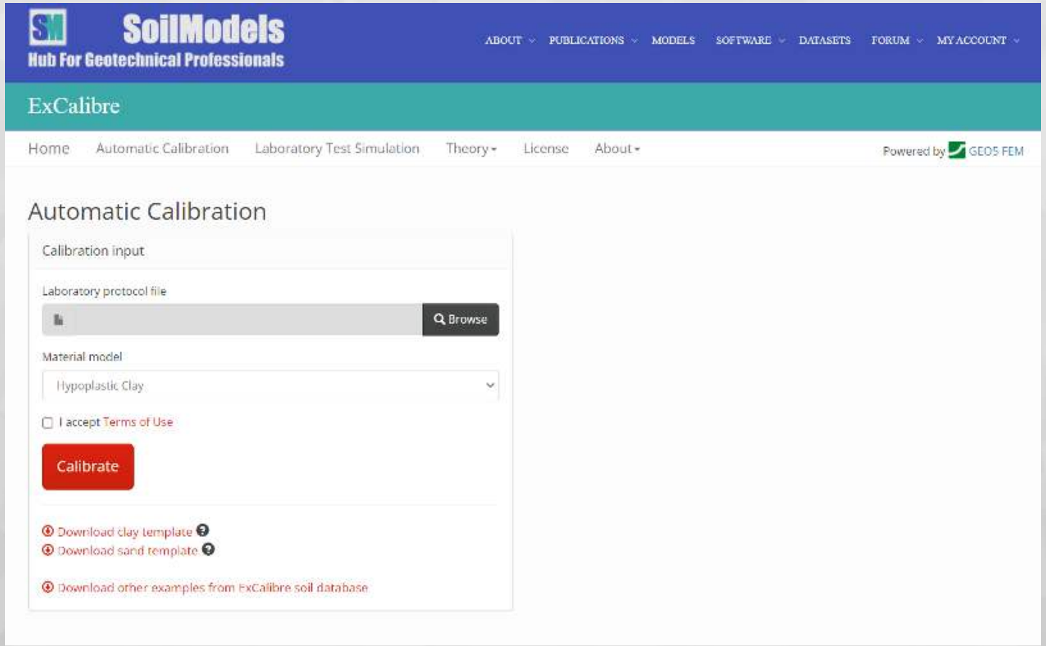
Calibration

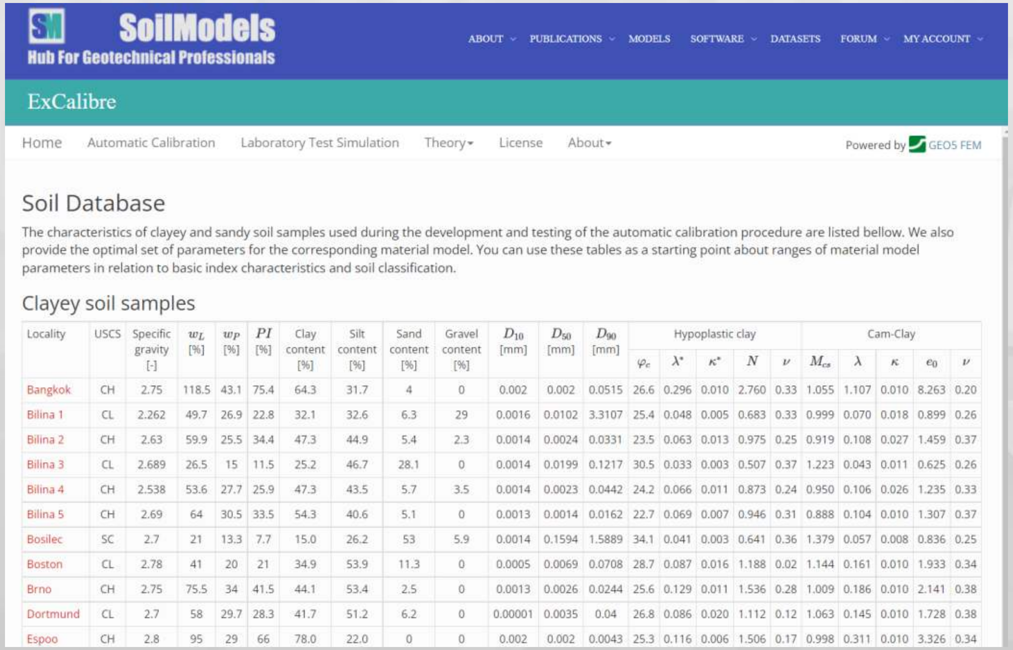
Mohr-Coulomb parameters

- Oedometer test
  - Primary loading branch → E_{oed}
  - Unloading-reloading branch → $E_{oed,ur}$
  - Both for chosen stress level
- Triaxial or direct shear test
  - Failure point → φ and c
- Use elasticity relations → E and ν
  - $E_{oed} = \frac{(1-\nu)}{(1+\nu)(1-2\nu)}E$
  - $G = \frac{1}{2(1+\nu)}E$

Cam clay and hypo clay parameters

- Free online tool ExCalibre (CTU in Prague, Charles university)
  - Automatic calibration for Cam clay and hypoplasic clay
  - https://soilmodels.com/excalibre-en/
  - Useful database of material parameters for various soils





Soil Database

The characteristics of clayey and sandy soil samples used during the development and testing of the automatic calibration procedure are listed bellow. We also provide the optimal set of parameters for the corresponding material model. You can use these tables as a starting point about ranges of material model parameters in relation to basic index characteristics and soil classification.

Clayey soil samples

Locality	USCS	Specific gravity [-]	w_L [%]	w_P [%]	PI [%]	Clay content [%]	Silt content [%]	Sand content [%]	Gravel content [%]	D_{10} [mm]	D_{50} [mm]	D_{90} [mm]	Hypoplastic clay					Cam-Clay				
													φ_c	λ^*	κ^*	N	ν	M_{cs}	λ	κ	e_0	ν
Bangkok	CH	2.75	118.5	43.1	75.4	64.3	31.7	4	0	0.002	0.002	0.0515	26.6	0.296	0.010	2.760	0.33	1.055	1.107	0.010	8.263	0.20
Bilina 1	CL	2.262	49.7	26.9	22.8	32.1	32.6	6.3	29	0.0016	0.0102	3.3107	25.4	0.048	0.005	0.683	0.33	0.999	0.070	0.018	0.899	0.26
Bilina 2	CH	2.63	59.9	25.5	34.4	47.3	44.9	5.4	2.3	0.0014	0.0024	0.0331	23.5	0.063	0.013	0.975	0.25	0.919	0.108	0.027	1.459	0.37
Bilina 3	CL	2.689	26.5	15	11.5	25.2	46.7	28.1	0	0.0014	0.0199	0.1217	30.5	0.033	0.003	0.507	0.37	1.223	0.043	0.011	0.625	0.26
Bilina 4	CH	2.538	53.6	27.7	25.9	47.3	43.5	5.7	3.5	0.0014	0.0023	0.0442	24.2	0.066	0.011	0.873	0.24	0.950	0.106	0.026	1.235	0.33
Bilina 5	CH	2.69	64	30.5	33.5	54.3	40.6	5.1	0	0.0013	0.0014	0.0162	22.7	0.069	0.007	0.946	0.31	0.888	0.104	0.010	1.307	0.37
Bosilec	SC	2.7	21	13.3	7.7	15.0	26.2	53	5.9	0.0014	0.1594	1.5889	34.1	0.041	0.003	0.641	0.36	1.379	0.057	0.008	0.836	0.25
Boston	CL	2.78	41	20	21	34.9	53.9	11.3	0	0.0005	0.0069	0.0708	28.7	0.087	0.016	1.188	0.02	1.144	0.161	0.010	1.933	0.34
Brno	CH	2.75	75.5	34	41.5	44.1	53.4	2.5	0	0.0013	0.0026	0.0244	25.6	0.129	0.011	1.536	0.28	1.009	0.186	0.010	2.141	0.38
Dortmund	CL	2.7	58	29.7	28.3	41.7	51.2	6.2	0	0.00001	0.0035	0.04	26.8	0.086	0.020	1.112	0.12	1.063	0.145	0.010	1.728	0.38
Espoo	CH	2.8	95	29	66	78.0	22.0	0	0	0.002	0.002	0.0043	25.3	0.116	0.006	1.506	0.17	0.998	0.311	0.010	3.326	0.34

Summary

- Mohr-Coulomb model – linear elastic with strength criterion
  - Suitable for strength-critical applications
  - Pay attention to E and ν
  - Depth-dependent stiffness
- Models of critical state
  - Stiffness increases with stress
  - Formulated for compressive σ_m - potential problems on terrain
  - ExCalibre web app at soilmodels.com
- OED and TRIAX laboratory tests
  - Represent material point
  - Easily simulated

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GEO5

Thank you for your attention

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