# Verification Analysis of Spread Footing

Program: Spread Footing
File: Demo_vm_en_02.gpa

In this verification manual you will find a hand-made verification analysis of bearing capacity of a spread footing for drained conditions in a permanent design situation. Results of the hand-made calculations are compared with the results from the GEO5 – Spread Footing program.

## Terms of Reference:

In Figure 1, an example of a centric spread footing is shown. The spread footing has a width $h = 1.80\ m$, a length $l = 2.20\ m$ and a height $t = 0.40\ m$. The footing bottom is in a depth $d = 1.20\ m$. The earth body is formed of silty sand (SM) and its terrain is adjusted in a $\beta = 7°$ inclination. The properties of soil (effective values) are shown in Table 1. In Figure 2, a load acting on the spread footing is shown and its values are in Table 2. The spread footing is covered with soil of unit weight $\gamma_z = 20.00\ kN/m^3$ after the realization. The spread footing is made from reinforced concrete with unit weight $\gamma_c = 23.00 kN/m^3$. The dimensions of the column on the spread footing are $c_x = 0.40\ m$ and $c_y = 0.40\ m$. The bearing capacity is calculated using the standard approach. The verification methodology is done according to EC 1997 with design approach 2 – reduction of actions and resistances. In this example the self-weight in favour $(\gamma_G = 1.00)$ and disfavour $(\gamma_G = 1.35)$ is calculated.

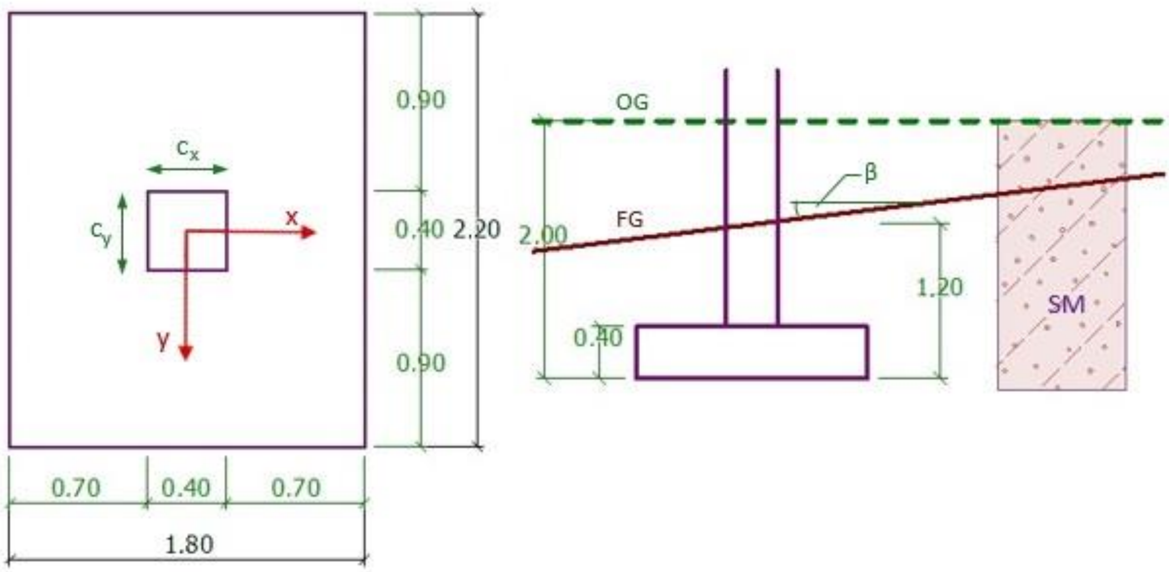


*Figure 1 Top view and cross-section in x of spread footing*

| Soil | Unit weight $\gamma$ [kN/m$^3$] | Saturated unit weight $\gamma_{sat}$ [kN/m$^3$] | Angle of internal friction $\varphi_{ef}$ [°] | Cohesion of soil $c_{ef}$ [kPa] | Poisson's ration $\nu$ [-] |
|---|---|---|---|---|---|
| SM | 17.50 | 17.50 | 31.50 | 0.00 | 0.35 |

*Table 1 Soil properties – characteristic effective values*

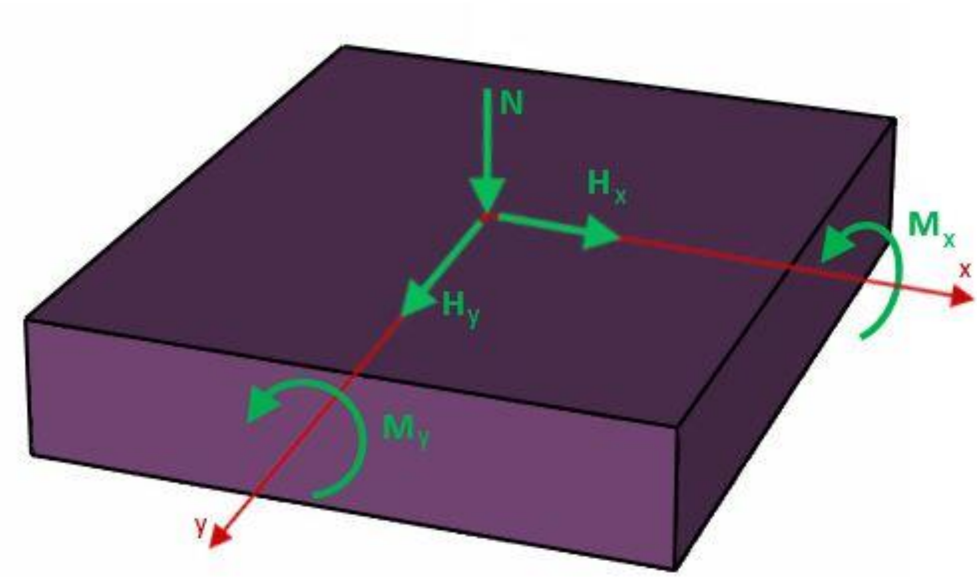


*Figure 2 Load on the spread footing*

| Design value | $N\,[kN]$ | $H_x\,[kN]$ | $H_y\,[kN]$ | $M_x\,[kNm]$ | $M_y\,[kNm]$ |
|---|---|---|---|---|---|
| Yes | 910.00 | 0.00 | 120.00 | 200.00 | 0.00 |

*Table 2 Values of the load*

## 1. The Calculation with Self-Weight acting Favourably

**Calculation of the effective area of the spread footing.** An eccentric load acts on the spread footing. Therefore, the effective area of the spread footing has to be calculated.

- Characteristic weight of the spread footing:
  $G = b \cdot h \cdot t \cdot \gamma_c = 1.80 \cdot 2.20 \cdot 0.40 \cdot 23.00 = 36.432\ kN$

- Characteristic weight of the overburden:

$$Z = [(d-t)\cdot b\cdot h-(d-t)\cdot c_x\cdot c_y]\cdot \gamma_z = [(1.20-0.40)\cdot 1.80\cdot 2.20-(1.20-0.40)\cdot 0.40\cdot 0.40]\cdot 20.00$$

$$Z = 60.80\ kN$$

- Calculation of the load eccentricity at the footing bottom $(\gamma_G = 1.00)$:

$$e_y = \frac{M_x + H_y \cdot t}{N + Z\cdot \gamma_G + G\cdot \gamma_G} = \frac{200.00 + 120.00\cdot 0.40}{910.00 + 60.80\cdot 1.00 + 36.432\cdot 1.00} = 0.246\ m$$

In the program, eccentricity is calculated as a ratio.

$$e_{y,ratio} = \frac{e_y}{l} = \frac{0.246}{2.200} = 0.112$$

- Verification of the load eccentricity:

$e_{x,ratio} = 0.000 \quad \le \quad 0.333 = e_{alw}$ - max. eccentricity in direction of the base length

$e_{y,ratio} = 0.112 \quad \le \quad 0.333 = e_{alw}$ - max. eccentricity in direction of the base width

$e_{t,ratio} = \sqrt{e^2_{x.ratio} + e^2_{y,ratio}}$ - max. overall eccentricity

$e_{t,ratio} = 0.112 \quad \le \quad 0.333 = e_{alw}$

The eccentricity of the load is **SATISFACTORY.**

**Result from the GEO5 – Spread Footing program:**

$e_{x,ratio} = 0.000 \quad \le \quad 0.333 = e_{alw}$

$e_{y,ratio} = 0.112 \quad \le \quad 0.333 = e_{alw}$

$e_{t,ratio} = 0.112 \quad \le \quad 0.333 = e_{alw}$**, SATISFACTORY**

- Calculation of the effective area of the spread footing (a rectangular shape of the effective area is assumed):

$$b_{eff} = b = 1.80\ m$$

$$l_{eff} = l - 2\cdot e = 2.20 - 2\cdot 0.246 = 1.708\ m$$

$$A_{eff} = b_{eff}\cdot l_{eff} = 1.80\cdot 1.708 = 3.074\ m^2$$

**Calculation of the vertical bearing capacity.**

A standard analysis proposed by J. Brinch Hansen is used.

- Coefficients of bearing capacity:

$$N_q = tg^2\left(45+\frac{\varphi}{2}\right)\cdot e^{\pi\cdot tg(\varphi)} = tg^2\left(45+\frac{31.5}{2}\right)\cdot e^{\pi\cdot tg(31.5)} = 21.861$$

$$N_c = (N_d - 1)\cdot \cot g(\varphi) = (21.861-1)\cdot \cot g(31.5) = 34.042$$

$$N_\gamma = 1.5\cdot (N_d - 1)\cdot tg(\varphi) = 1{,}5\cdot (21.861-1)\cdot tg(31.5) = 19.175$$

- Coefficients of the shape of the foundation:
  Due to the eccentricity, the following situation occurred: $l_{eff} < b_{eff}$. Therefore, $b = l_{eff}$ and $l = b_{eff}$ were used.

$$s_q = 1+\frac{b}{l}\cdot\sin(\varphi) = 1+\frac{1.708}{1.800}\cdot\sin(31.5) = 1.496$$

$$s_c = 1+0.2\cdot\frac{b}{l} = 1+0.2\cdot\frac{1.708}{1.800} = 1.190$$

$$s_\gamma = 1-0.3\cdot\frac{b}{l} = 1-0.3\cdot\frac{1.708}{1.800} = 0.715$$

- Coefficients of influence of the foundation depth:

$$d_q = 1+0.1\cdot\sqrt{\frac{d}{b}\cdot\sin(2\cdot\varphi)} = 1+0.1\cdot\sqrt{\frac{1.200}{1.708}\cdot\sin(2\cdot 31.5)} = 1.079$$

$$d_c = 1+0.1\cdot\sqrt{\frac{d}{b}} = 1+0.1\cdot\sqrt{\frac{1.200}{1.708}} = 1.084$$

$$d_\gamma = 1.000$$

- Coefficients of influence of the slope load:
  $\delta$ - angle of deviation of the resultant force from the vertical direction

$$\delta = arctg\left(\frac{H_y}{N+Z\cdot\gamma_G+G\cdot\gamma_G}\right) = arctg\left(\frac{120}{910+60.80\cdot 1.00+36.432\cdot 1.00}\right) = 6.794^\circ$$

$$i_q = i_c = i_\gamma = (1-tg(\delta))^2 = (1-tg(6.794))^2 = 0.776$$

- Coefficients of slope of the footing bottom:
  $\alpha$ - slope of the footing bottom
  $\alpha = 0.000^\circ$
  $b_q = (1-\alpha\cdot tg(\varphi))^2 = (1-0\cdot tg(31.5))^2 = 1.000$

$$b_c = b_q - \frac{(1-b_q)}{N_c} \cdot tg(\varphi) = 1 - \frac{(1-1)}{34.042} \cdot tg(31.5) = 1.000$$

$$b_\gamma = b_q = 1.000$$

- Coefficients of influence of the slope of the terrain:
  $\beta$ - slope of the terrain
  $\beta = 7.000°$

$$g_q = g_\gamma = (1 - 0.5 \cdot tg(\beta))^5 = (1 - 0.5 \cdot tg(7.0))^5 = 0.728$$

$$g_c = 1 - \frac{2 \cdot \beta}{\pi + 2} = 1 - \frac{2 \cdot \left(7.000 \cdot \frac{\pi}{180}\right)}{\pi + 2} = 0.952$$

- Equivalent uniform load accounting for the influence of the foundation depth:
  $q_0 = \gamma_1 \cdot d = 17.5 \cdot 1.20 = 21.00\ kPa$

- Calculation of the vertical bearing capacity and its reduction by coefficient $\gamma_{RV} = 1.40$ :

$$R_d = c \cdot N_c \cdot s_c \cdot d_c \cdot i_c \cdot b_c \cdot g_c + q_0 \cdot N_q \cdot s_q \cdot d_q \cdot i_q \cdot b_q \cdot g_q + \gamma \cdot \frac{b}{2} \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma \cdot i_\gamma \cdot c_\gamma \cdot b_\gamma \cdot g_\gamma$$

$$R_d = 0.0 + 21.0 \cdot 21.861 \cdot 1.496 \cdot 1.079 \cdot 0.776 \cdot 1.0 \cdot 0.728 + 17.50 \cdot \frac{1.708}{2} \cdot 19.175 \cdot 0.715 \cdot 1.0 \cdot 0.776 \cdot 1.0 \cdot 0.728$$

$$R_d = 0.0 + 418.635 + 115.753$$

$$R_d = 534.388\ kPa$$

$$\frac{R_d}{\gamma_{RV}} = \frac{534.388}{1.40} = 381.706\ kPa$$

**Result from the GEO5 – Spread Footing program:** $R_d = 381.92\ kPa$

- Extreme contact stress at the footing bottom:

$$\sigma = \frac{N + Z \cdot \gamma_G + G \cdot \gamma_G}{A_{eff}} = \frac{910.00 + 60.80 \cdot 1.00 + 36.432 \cdot 1.00}{3.074} = 327.662\ kPa$$

**Result from the GEO5 – Spread Footing program:** $\sigma = 327.70\ kPa$

- Usage:

$$V_u = \frac{\sigma}{R_d} \cdot 100 = \frac{327.662}{381.706} \cdot 100 = 85.8\ \%$$ , SATISFACTORY

**Result from the GEO5 – Spread Footing program:** $V_u = 85.8\ \%$ **, SATISFACTORY**

**Calculation of the horizontal bearing capacity.**

The earth resistance is taken as pressure at rest.

- Coefficient of earth pressure at rest in a drained soil:
  $K_0 = 1 - \sin(\varphi_d) = 1 - \sin(31.5) = 0.4775$

- Pressure at rest in the axis on the upper and lower edge of the spread footing:
  $\sigma_{01} = \gamma \cdot (d - t) \cdot K_0 = 17.50 \cdot (1.20 - 0.40) \cdot 0.4775 = 6.6850\ kPa$

  $\sigma_{z2} = \gamma \cdot d \cdot K_0 = 17.50 \cdot 1.20 \cdot 0.4775 = 10.0275\ kPa$

- Value of earth resistance:
  $$S_{pd} = \left[\frac{1}{2} \cdot (\sigma_{02} - \sigma_{01}) \cdot t + \sigma_{01} \cdot t\right] \cdot b = \left[\frac{1}{2} \cdot (10.0275 - 6.6850) \cdot 0.40 + 6.6850 \cdot 0.40\right] \cdot 1.80 = 6.017\ kN$$

- Resultant vertical force on the footing bottom:
  $Q = V + Z + G = 910.00 + 60.80 + 36.432 = 1007.232\ kN$

- Horizontal bearing capacity and its reduction by coefficient $\gamma_{RH} = 1.10$ :
  $a_d \cdot A_{eff}$ - is excluded for drained conditions (according to EC 1997)

$$R_{dh} = \frac{Q \cdot tg(\varphi_d) + a_d \cdot A_{eff} + S_{pd}}{\gamma_{RH}}$$

$$R_{dh} = \frac{Q \cdot tg(\varphi_d) + S_{pd}}{\gamma_{RH}} = \frac{1007.232 \cdot tg(31.5) + 6.017}{1.10} = 566.591\ kN$$

**Result from the GEO5 – Spread Footing program:** $R_{dh} = 566.59\ kPa$

- Usage:
  $$V_u = \frac{H}{R_{dh}} \cdot 100 = \frac{120.000}{566.591} \cdot 100 = 21.2\ \%$$ , SATISFACTORY

**Result from the GEO5 – Spread Footing program:** $V_u = 21.2\ \%$ **, SATISFACTORY**

# 2. The Calculation with Self-Weight Acting Unfavourably

**Calculation of the effective area.** The characteristic weights of the spread footing and the overburden are the same as in the calculation with self-weight in favor.

- Characteristic weight of the spread footing:
  $G = 36.432\ kN$

- Characteristic weight of the overburden:
  $Z = 60.80\ kN$

- Calculation of the load eccentricity at the footing bottom $(\gamma_G = 1.35)$:
  $$e_y = \frac{M_x + H_y \cdot t}{N + Z \cdot \gamma_G + G \cdot \gamma_G} = \frac{200.00 + 120.00 \cdot 0.40}{910.00 + 60.80 \cdot 1.35 + 36.432 \cdot 1.35} = 0.238\ m$$

  In the program, eccentricity is calculated as a ratio.
  $$e_{y,pom} = \frac{e_y}{l} = \frac{0.238}{2.200} = 0.108$$

- Verification of the load eccentricity:
  $e_{x,pom} = 0.000 \quad \leq \quad 0.333 = e_{alw}$ - max. eccentricity in direction of the base length

  $e_{y,pom} = 0.108 \quad \leq \quad 0.333 = e_{alw}$ - max. eccentricity in direction of the base width

  $e_{t,pom} = \sqrt{e^2_{x.pom} + e^2_{y,pom}}$ - max. overall eccentricity

  $e_{t,pom} = 0.108 \quad \leq \quad 0.333 = e_{alw}$

  The eccentricity of the load is **SATISFACTORY.**

  **Result from the GEO5 – Spread Footing program:**
  The GEO5 – Spread Footing program shows the verification of a load that causes greater eccentricity. In this case a load with self-weight in favour is verified.

- Calculation of the effective area of the spread footing (a rectangular shape of the effective area is assumed):
  $b_{eff} = b = 1.80\ m$

  $l_{eff} = l - 2 \cdot e = 2.20 - 2 \cdot 0.238 = 1.724\ m$

  $A_{eff} = b_{eff} \cdot l_{eff} = 1.80 \cdot 1.724 = 3.103\ m^2$

**Calculation of the vertical bearing capacity.**

- Coefficients of bearing capacity:

$$N_q = tg^2\left(45 + \frac{\varphi}{2}\right) \cdot e^{\pi \cdot tg(\varphi)} = tg^2\left(45 + \frac{31.5}{2}\right) \cdot e^{\pi \cdot tg(31.5)} = 21.861$$

$$N_c = (N_d - 1) \cdot \cot g(\varphi) = (21.861 - 1) \cdot \cot g(31.5) = 34.042$$

$$N_\gamma = 1.5 \cdot (N_d - 1) \cdot tg(\varphi) = 1.5 \cdot (21.861 - 1) \cdot tg(31.5) = 19.175$$

- Coefficients of the shape of the foundation:
  Due to the eccentricity, the following situation occurred: $l_{eff} < b_{eff}$. Therefore, $b = l_{eff}$ and $l = b_{eff}$ were used.

$$s_q = 1 + \frac{b}{l} \cdot \sin(\varphi) = 1 + \frac{1.724}{1.800} \cdot \sin(31.5) = 1.500$$

$$s_c = 1 + 0.2 \cdot \frac{b}{l} = 1 + 0.2 \cdot \frac{1.724}{1.800} = 1.192$$

$$s_\gamma = 1 - 0.3 \cdot \frac{b}{l} = 1 - 0.3 \cdot \frac{1.724}{1.800} = 0.713$$

- Coefficients of influence of the foundation depth:

$$d_q = 1 + 0.1 \cdot \sqrt{\frac{d}{b} \cdot \sin(2 \cdot \varphi)} = 1 + 0.1 \cdot \sqrt{\frac{1.200}{1.724} \cdot \sin(2 \cdot 31.5)} = 1.079$$

$$d_c = 1 + 0.1 \cdot \sqrt{\frac{d}{b}} = 1 + 0.1 \cdot \sqrt{\frac{1.200}{1.724}} = 1.083$$

$$d_\gamma = 1.000$$

- Coefficients of influence of the slope of the load:
  $\delta$ - angle of deviation of the resultant force from the vertical direction

$$\delta = arctg\left(\frac{H_y}{N + Z \cdot \gamma_G + G \cdot \gamma_G}\right) = arctg\left(\frac{120}{910 + 60.80 \cdot 1.35 + 36.432 \cdot 1.35}\right) = 6.574°$$

$$i_q = i_c = i_\gamma = (1 - tg(\delta))^2 = (1 - tg(6.574))^2 = 0.783$$

- Coefficients of the slope of the footing bottom:
  $\alpha$ -slope of the footing bottom
  $\alpha = 0.000°$

$$b_q = (1 - \alpha \cdot tg(\varphi))^2 = (1 - 0 \cdot tg(31.5))^2 = 1.000$$

$$b_c = b_q - \frac{(1 - b_q)}{N_c} \cdot tg(\varphi) = 1 - \frac{(1-1)}{34.042} \cdot tg(31.5) = 1.000$$

$$b_\gamma = b_q = 1.000$$

- Coefficients of influence of the slope of the terrain:
  $\beta$ - slope of the terrain
  $\beta = 7.000°$

$$g_q = g_\gamma = (1 - 0.5 \cdot tg(\beta))^5 = (1 - 0.5 \cdot tg(7.0))^5 = 0.728$$

$$g_c = 1 - \frac{2 \cdot \beta}{\pi + 2} = 1 - \frac{2 \cdot \left(7.000 \cdot \frac{\pi}{180}\right)}{\pi + 2} = 0.952$$

- Equivalent uniform load accounting for the influence of the foundation depth:
  $q_0 = \gamma_1 \cdot d = 17.50 \cdot 1.20 = 21.00\ kPa$

- Calculation of the vertical bearing capacity and its reduction by coefficient $\gamma_{RV} = 1.40$ :

$$R_d = c \cdot N_c \cdot s_c \cdot d_c \cdot i_c \cdot b_c \cdot g_c + q_0 \cdot N_q \cdot s_q \cdot d_q \cdot i_q \cdot b_q \cdot g_q + \gamma \cdot \frac{b}{2} \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma \cdot i_\gamma \cdot c_\gamma \cdot b_\gamma \cdot g_\gamma$$

$$R_d = 0.0 + 21.0 \cdot 21.861 \cdot 1.500 \cdot 1.079 \cdot 0.783 \cdot 1.0 \cdot 0.728 + 17.50 \cdot \frac{1.724}{2} \cdot 19.175 \cdot 0.713 \cdot 1.0 \cdot 0.783 \cdot 1.0 \cdot 0.728$$

$$R_d = 0.0 + 423.541 + 117.561$$

$$R_d = 541.102\ kPa$$

$$\frac{R_d}{\gamma_{RV}} = \frac{541.102}{1.40} = 386.501\ kPa$$

**Result from the GEO5 – Spread Footing program:** $R_d = 386.61\ kPa$

- Extreme contact stress at the footing bottom:

$$\sigma = \frac{N + Z \cdot \gamma_G + G \cdot \gamma_G}{A_{eff}} = \frac{910.00 + 60.80 \cdot 1.35 + 36.432 \cdot 1.35}{3.103} = 335.567\ kPa$$

**Result from the GEO5 – Spread Footing program:** $\sigma = 335.61\ kPa$

- Usage:

$$V_u = \frac{\sigma}{R_d} \cdot 100 = \frac{335.567}{386.501} \cdot 100 = 86.8\ \%$$, SATISFACTORY

**Calculation of the parameters of a slip surface below the foundation.** The parameters of a slip surface are calculated according to Prandtl's theory.

- Depth of the slip surface:

$$z_{sp} = \frac{b}{2} \cdot \frac{\cos(\varphi)}{\cos\left(45 + \frac{\varphi}{2}\right)} \cdot e^{\left(\frac{\pi}{4} + \frac{arc(\varphi)}{2}\right) \cdot tg(\varphi)} = \frac{1.80}{2} \cdot \frac{\cos(31.5)}{\cos\left(45 + \frac{31.5}{2}\right)} \cdot e^{\left(\frac{\pi}{4} + \frac{arc(31.5)}{2}\right) \cdot tg(31.5)} = 3.008\ m$$

**Result from the GEO5 – Spread Footing program:** $z_{sp} = 3.01\ m$

- Length of the slip surface:

$$l_{sp} = \frac{b}{2} \cdot \left[1 + 2 \cdot tg\left(45 + \frac{\varphi}{2}\right) \cdot e^{\frac{\pi}{2} \cdot tg(\varphi)}\right] = \frac{1.80}{2} \cdot \left[1 + 2 \cdot tg\left(45 + \frac{31.5}{2}\right) \cdot e^{\frac{\pi}{2} \cdot tg(31.5)}\right] = 9.316\ m$$

**Result from the GEO5 – Spread Footing program:** $z_{sp} = 9.31\ m$